M.Sc
Part - II

Paper IV - VI
Optional - X

OPERATIONS

MATHEMATICS

RESEARCH

Original Notes

Traders

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SECTION-I

Q: Write a note on linear programming problems? (LPP)

Sol: The general form of LPP is given below.

$$Z = CX \rightarrow (1)$$

Subject to the constraint

$$AX \ (\leq, =, \geq) \ b \rightarrow (2)$$

$$X \geq 0 \rightarrow (3)$$

Here;

X is a $n \times 1$ column matrix.

A is a $m \times n$ matrix.

b is a $m \times 1$ column matrix.

C is a $1 \times n$ row matrix.

So,

$$X = (x_1, x_2, \ldots\ldots x_n)^t$$

$$C = (c_1, c_2, \ldots\ldots c_n)$$

$$b = (b_1, b_2, \ldots\ldots b_m)^t$$

$$A = \begin{pmatrix} a_{11} & a_{12} \ldots\ldots b_{1n} \\ a_{21} & a_{22} \ldots\ldots a_{2n} \\ \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \\ a_{m1} & a_{m2} \ldots\ldots a_{mn} \end{pmatrix}$$

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The expression in eq (1) is called as objective function. The inequalities in eq (2) are called as constraints and the

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relation in eq. ③ are called non-negative restrictions.

The relations in eq. ①, ② and ③ can be written explicitly as optimize

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

Subject to

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \ (\leq, =, \geq)\ b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \ (\leq, =, \geq)\ b_2$$
$$\cdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \ (\leq, =, \geq)\ b_m$$

where,

$$x_1, x_2, x_3 \cdots x_n \geq 0$$

Sometimes the constraints are not given in the equation form. In this case slack variables and surplus variables can be used to change the inequalities into the equation form.

Consider,

$$2x_1 + x_2 + 3x_3 \leq 5$$

Then, a non-negative variable say x_4 can be added to give.

$$2x_1 + x_2 + 3x_3 + x_4 = 5$$

where x_4 is a slack variable.

If the constraint is of the form

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$x_1 + 5x_2 + x_3 \geq 10$ ... x_4

Then, a non-negative variable ↑ can be subtracted to give

$x_1 + 5x_2 + x_3 - x_4 = 10$

where x_4 is called as surplus variable.

Solutions Of An LPP:-

Consider the LPP, optimize

$Z = CX \rightarrow (1)$

Subject to

$AX \ (\leq, =, \geq)\ b \rightarrow (2)$

$X \geq 0 \rightarrow (3)$

The condition on "b" $b \geq 0$.

Convert the constraints into equations by adding slacks or subtracting surplus variables. Therefore eq. (2) gives

$AX = b \rightarrow (4)$

A vector $X = (x_1, x_2, \ldots, x_n)^t$ which satisfies (3) and (4) eqs. is called a feasible solution.

Basic Solution:-

Consider the following system

$x_1 + x_2 + x_3 = 3$

$x_1 - x_2 + 2x_3 = 2$

No. of variables C No. of variables − No. of constraints.

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There will be ${}^3C_1$ basic solution of the given system.

Now, we will find the all basic solutions.

Putting

$$x_1 = 0$$

Then:

$$x_2 + x_3 = 3 \rightarrow (*)$$

$$-x_2 + 2x_3 = 2$$

Adding

$$3x_3 = 5$$

$$x_3 = \frac{5}{3}$$

$(*) \Rightarrow x_2 + \frac{5}{3} = 3$

$$\Rightarrow x_2 = \frac{4}{3}$$

So,

Sol. is $(0, \frac{4}{3}, \frac{5}{3})$

Putting:-

$$x_2 = 0$$

$\Rightarrow$

$$x_1 + x_3 = 3 \rightarrow (*)'$$

$$x_1 + 2x_3 = 2$$

Subtracting

$$-x_3 = 1$$

$$x_3 = -1$$

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From ⊛'

$x_1 = 4$

So,

Sol is $(4, 0, -1)$

Now, putting

$x_3 = 0$

$\Rightarrow$

$x_1 + x_2 = 3 \rightarrow$ ⊛''

$x_1 - x_2 = 2$

Adding

$2x_1 = 5$

$x_1 = \frac{5}{2}$.

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⊛'' $\Rightarrow$

$x_2 = \frac{1}{2}$

So,

Sol is $(\frac{5}{2}, \frac{1}{2}, 0)$

All these three solutions are basic solutions.

Also the solutions are feasible but Sol. (2) is not feasible because it involves x_3 is -ive.

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Primal Simplex Method OR The Simplex Method OR Regular Simplex Method :-

Standard form of the Simplex Method :-
Properties of this form are as follows.

i) First of all we convert all the constraints into equations by adding a slack variable or by subtracting a surplus variable in the respective constraint to make it equation.

ii) All variables are non-negative.

iii) The objective function may be extremize. (maximize or minimize).

iv) Each constraint in the linear programming model must have a slack variable with non-negative R.H.S.

Working Rules:-

i) Obtain physible feasible starting solution from the standard form.

ii) Select the entering variables (which is one of the non-basic variables) by using optimality conditions.

Optimality Conditions:-

The entering variable in maximization (minimization) is the non-basic variable with the most -ive

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(+ive) coefficient in the objective Z-equation. These optimum is reached when all the non-basic coefficient is the Z-equation are non-negative (non-positive).

(iii) Select the leaving variable (which is the one of the current basic variable) by using the feasibility conditions.

FEASIBILITY Condition:-

The leaving variable is the current basic variable having the smallest intercepts (minimum ratio). with strickly +ive denomerator is the direction of the entering variable.

(iv) Obtain a new basic solution by using Gauss Jorden ~~method~~ elimination method.

If all the non-basic variables have the non-negative (non-positive) coefficient in the Z-equation in case of maximization (minimization), then solution coulumn gives the required optimize solution.

Otherwise next alteration and continue the above process until the optimum solution is reached.

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Z کے Variables ہمیشہ جب تک وہ خود نہ کہے non-basic ہوتے ہیں۔

Q1:- Use simplex method technique to solve the following problems.

Max.

$$Z = 5x_1 + 4x_2$$

Subjected to

$$3x_1 + 2x_2 \leq 12$$
$$x_1 + 2x_2 \leq 6$$
$$x_2 \leq 2$$
$$-x_1 + x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

Sol:-

Give problem is standard form is

Max.

$$Z = 5x_1 + 4x_2$$

Z-eq. is

$$Z - 5x_1 - 4x_2 = 0$$

Subject to

$$3x_1 + 2x_2 + x_3 = 12$$
$$x_1 + 2x_2 + x_4 = 6$$
$$x_2 + x_5 = 2$$
$$-x_1 + x_2 + x_6 = 1$$

where;

$$x_3, x_4, x_5, x_6 \geq 0$$

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The given problem is tabular form is.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	-5	-4	0	0	0	0	0	
← x_3	(3) P.E	2	1	0	0	0	12	$\frac{12}{3}=4$
x_4	1	2	0	1	0	0	6	$\frac{6}{1}=6$
x_5	0	1	0	0	1	0	2	
x_6	-1	1	0	0	0	1	1	

Here (3) is called pivot element and x_1 is entering and x_3 is leaving element.

Rough Work.

$-4+\frac{10}{3}=\frac{-12+10}{3}$

$-5 \quad -4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$

$5 \quad 10/3 \quad 5/3 \quad 0 \quad 0 \quad 0 \quad 20$

$0 \quad -2/3 \quad 5/3 \quad 0 \quad 0 \quad 0 \quad 20$

$2-\frac{2}{3}=\frac{6-2}{3}$

$1 \quad 2 \quad 0 \quad 1 \quad 0 \quad 0 \quad 6$

$\pm 1 \quad \pm 2/3 \quad \pm 1/3 \quad 0 \quad 0 \quad 0 \quad \pm 4$

$1+\frac{2}{3}=\frac{3+2}{3}$

$0 \quad 4/3 \quad -1/3 \quad \phi \quad 0 \quad 0 \quad 2$

$-1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1$

$1 \quad 2/3 \quad 1/3 \quad 0 \quad 0 \quad 0 \quad 4$

$0 \quad 5/3 \quad 1/3 \quad 0 \quad 0 \quad 1 \quad 5$

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio.
Z	0	-2/3	5/3	0	0	0	20	
x_1	1	2/3	1/3	0	0	0	4	$\frac{4}{2/3} = 6$
← x_4	0	(4/3) P.E	-1/3	1	0	0	2	$\frac{2}{4/3} = \frac{3}{2}$
x_5	0	1	0	0	1	0	2	$\frac{2}{1} = 2$
x_6	0	5/3	1/3	0	0	1	5	$\frac{5}{\frac{5}{3}} = 3$

$\frac{4}{3}$ is pivot element and x_2, x_3 are entering and leaving elements.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio.
Z	0	0	3/2	1/2	0	0	21	
x_1	1	0	1/2	-1/2	0	0	3	
x_2	0	1	-1/4	3/4	0	0	3/2	
x_5	0	0	1/4	-3/4	1	0	1/2	
x_6	0	0	3/4	-5/4	0	1	5/2	

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(ii)

This is the optimal table and optimal solution is

$Z = 21$, $x_1 = 3$; $x_2 = \frac{3}{2}$,

∴ Rough Work :-

$(\frac{3}{4})(-\frac{1}{3}) = -\frac{1}{4}$

0	$-\frac{2}{3}$	$\frac{5}{3}$	0	0	0	20	$\frac{5}{3} - \frac{1}{6}$, $10 - 1$, 9, 6
0	$\frac{2}{3}$	$-\frac{1}{6}$	$\frac{1}{2}$	0	0	1	
0	0	3/2	1/2	0	0	21	$(-\frac{1}{3})(\frac{2}{3})$
1	2/3	1/3	0	0	0	4	$(\frac{2}{3})(\frac{2}{3})$
± 0	± 2/3	∓ 1/6	± 1/2	0	0	± 1	$\frac{1}{3} + \frac{1}{6} = \frac{2+1}{6}$
1	0	1/2	−1/2	0	0	3	
0	1	0	0	1	0	2	$2 - \frac{3}{2}$
0	± 1	∓ 1/4	± 3/4	± 0	± 0	± 3/2	$\frac{4-3}{2} = \frac{1}{2}$
0	0	1/4	−3/4	1	0	1/2	
0	5/3	1/3	0	0	1	5	
0	± 5/3	∓ 5/12	± 3/4	0	0	± 5/2	
0	0	3/4	−5/4	0	1	5/2	

$\frac{1}{3} + \frac{5}{12}$

$= \frac{4+5}{12} = \frac{9}{12}$, $5 - \frac{5}{2} = \frac{10-5}{2} = \frac{5}{2}$

$= \frac{3}{4}$

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Q2:- Maximize

$$Z = 4x_1 + 3x_2$$

Subject to

$$2x_1 + 3x_2 \leq 6$$
$$-3x_1 + 2x_2 \leq 3$$
$$2x_2 \leq 5$$
$$2x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Sol:-

The given problem in standard form is.

Max.

$$Z = 4x_1 + 3x_2$$

Z-equation is.

$$Z - 4x_1 - 3x_2 = 0$$

Subject to

$$2x_1 + 3x_2 + x_3 = 6$$
$$-3x_1 + 2x_2 + x_4 = 3$$
$$2x_2 + x_5 = 5$$
$$2x_1 + x_2 + x_6 = 4$$

In tabular form is.

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	-4	-3	0	0	0	0	0	
x_3	2	3	1	0	0	0	6	$\frac{6}{2} = 3$
x_4	-3	2	0	1	0	0	3	
x_5	0	2	0	0	1	0	5	
← x_6	(2) P.E	1	0	0	0	1	4	$\frac{4}{2} = 2$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	0	-1	0	0	0	2	8	
← x_3	0	(2) P.E	1	0	0	-1	2	$\frac{2}{2} = 1$
x_4	0	7/2	0	1	0	3/2	9	$\frac{9}{7/2} = \frac{18}{7}$
x_5	0	2	0	0	1	0	5	$\frac{5}{2}$
x_1	1	1/2	0	0	0	1/2	2	$\frac{2}{1/2} = 4$

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	0	0	1/2	0	0	3/2	9	
x_2	0	1	1/2	0	0	-1/2	1	
x_4	0	0	-7/4	1	0	13/4	11/2	
x_5	0	0	-1	0	1	1	3	
x_1	1	0	-1/4	0	0	3/4	3/2	

This is the optimal table and optimal solution is:

$Z = 9, \quad x_1 = 3/2, \quad x_2 = 1$

∴

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Rough Work

-4	-3	0	0	0	0	0
4	2	0	0	0	2	8
0	-1	0	0	0	2	8

	2	3	1	0	0	0	6
	÷2	÷1	÷0	÷0	÷0	÷1	÷4
$2+\frac{3}{2}$	0	2	1	0	0	-1	2
	-3	2	0	1	0	0	3
$\frac{4+3}{2}=\frac{7}{2}$	3	3/2	0	0	0	3/2	6
	0	7/2	0	1	0	3/2	9

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0	−1	0	0	0	2	8
0	1	1/2	0	0	−1/2	1
0	0	1/2	0	0	3/2	9
0	7/2	0	1	0	3/2	9
0	∓7/2	∓7/4	0	0	∓7/4	∓7/2
0	0	−7/4	1	0	13/4	11/2
0	2	0	0	1	0	5
0	∓2	∓1	0	0	−1	∓2
0	0	−1	0	1	1	3
1	1/2	0	0	0	1/2	2
∓0	∓1/2	∓1/4	0	0	∓1/4	∓1/2
1	0	−1/4	0	0	3/4	3/2

$\frac{3}{2} + \frac{7}{4}$

$\frac{6+7}{4} = \frac{13}{4}$

$9 - \frac{7}{2}$

$\frac{18-7}{2}$

$\frac{1}{2} + \frac{1}{4}$

$\frac{2+1}{4}$

Q:- Minimize

$$Z = x_1 - 3x_2 - 2x_3$$

Subject to

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$-2x_1 + 4x_2 \leq 12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

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Sol:-

The given problem in standard form is

Minimize

$$Z = x_1 - 3x_2 + 2x_3$$

Z-equation is

$$Z - x_1 + 3x_2 - 2x_3 = 0$$

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Subjected to

$$3x_1 - x_2 + 2x_3 + x_4 = 7$$
$$-2x_1 + 4x_2 + x_5 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + x_6 = 10$$
$$x_i \geq 0 \quad i = 1, 2, \ldots 6.$$

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In tabular form:-

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	-1	3	2	0	0	0	0	
x_4	3	-1	2	1	0	0	7	
← x_5	-2	(4) P.E	0	0	1	0	12	$\frac{12}{4}=3$
x_6	-4	3	8	0	0	1	10	$\frac{10}{3}=3.3$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	1/2	0	2	0	-3/4	0	-9	
x_4	5/2	0	2	1	1/4	0	10	$\frac{10}{2}=5$
x_2	$-\frac{1}{2}$	1	0	0	1/4	0	3	
← x_6	-5/2	0	(8)	0	-3/4	1	1	$\frac{1}{8}$

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	9/8	0	0	0	-9/16	-1/4	-37/4	
	P.E							
← x_4	25/8	0	0	1	7/16	-1/4	39/4	
~~x_2~~	-1/2	1	0	0	1/4	0	3	
x_3	-5/16	0	1	0	-3/32	1/8	1/8	

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:: Rough Work :-

$-1 \quad 3 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1+\frac{3}{2}$

$\mp 3/2 \quad \pm 3 \quad \pm 0 \quad \pm 0 \quad \pm 3/4 \quad \pm 0 \quad \pm 9 \quad \frac{-2+3}{2}$

$1/2 \quad 0 \quad 2 \quad 0 \quad -3/4 \quad 0 \quad -9$

$3 \quad -1 \quad 2 \quad 1 \quad 0 \quad 0 \quad 7 \quad 3-1$

$-1/2 \quad 1 \quad 0 \quad 0 \quad 1/4 \quad 0 \quad 3 \quad \frac{6-1}{2}$

$5/2 \quad 0 \quad 2 \quad 1 \quad 1/4 \quad 0 \quad 10$

$-4 \quad 3 \quad 8 \quad 0 \quad 0 \quad 1 \quad 10 \quad -4+\frac{3}{2}$

$\mp 3/2 \quad \pm 3 \quad \pm 0 \quad \pm 0 \quad \pm 3/4 \quad \pm 0 \quad \pm 9 \quad \frac{-8+3}{2}$

$-5/2 \quad 0 \quad 8 \quad 0 \quad -3/4 \quad 1 \quad 1$

$\left(-\frac{5}{2}\right)\left(\frac{1}{8}\right) = -\frac{5}{16} \qquad \left(-\frac{3}{4}\right)\left(\frac{1}{8}\right)$

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$\frac{1}{2}$	0	2	0	$-3/4$	0	-9
$\pm\frac{5}{8}$	0	± 2	0	$\mp 3/16$	$\pm 1/4$	$\pm 1/4$
$9/8$	0	0	0	$-9/16$	$-1/4$	$-37/4$

$\frac{1}{2} + \frac{5}{8} = \frac{4+5}{8} = \frac{9}{8}$, $-\frac{3}{4} + \frac{3}{16} = \frac{-12+3}{16} = -\frac{9}{16}$

$-9 - \frac{1}{4} = \frac{-36-1}{4} = -\frac{37}{4}$

$2\frac{5}{8}$	0	2	1	$\frac{1}{4}$	0	10
$\mp\frac{5}{8}$	0	± 2	± 0	$\mp\frac{3}{16}$	$\pm\frac{1}{4}$	$\pm\frac{1}{4}$
$25/8$	0	0	1	$7/16$	$-1/4$	

$2\frac{5}{8} + \frac{5}{8} = \frac{20+5}{8} = \frac{25}{8}$; $\frac{1}{4} + \frac{3}{16} = \frac{4+3}{16} = 7/16$

$10 - \frac{1}{4} = \frac{40-1}{4} = 39/4$

$-\frac{1}{2}$ X 0 0 $\frac{1}{4}$ 0 0

$\left(\frac{8}{25}\right)\left(\frac{7}{16}\right) = \frac{7}{50}$; $\left(\frac{2\,8}{25}\right)\left(\frac{39}{4}\right) = \frac{78}{25}$

$\left(-\frac{1}{4}\right)\left(\frac{8}{25}\right) = -2/25$; $\left(\frac{9}{8}\right)\left(\frac{8}{25}\right) = \frac{9}{25}$

$\left(\frac{7}{50}\right)\left(\frac{9}{8}\right) = \frac{63}{400}$ $\quad \frac{63}{400} = \frac{9}{16}$

$1\frac{1}{4} = \frac{5}{4}$ $\quad \left(\frac{-2}{25}\right)\left(\frac{9}{2}\right)$

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$39\left(\frac{70}{25}\right)\left(\frac{9}{8}\right) = \frac{357}{100} - \frac{4}{25}$ $-\frac{1}{4} + \frac{18}{200} = -\frac{1}{4} + \frac{7}{100}$

$= \frac{-25+9}{100}$

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$\frac{9}{8}$	0	0	0	$-\frac{9}{16}$	$-\frac{1}{4}$	$-\frac{37}{4}$	$\frac{32}{25}$
$\pm\frac{9}{8}$	0	0	$\pm\frac{9}{25}$	$\pm\frac{63}{400}$	$-\frac{18}{200}$	$+\frac{357}{100}$	
0	0	0	$-9/25$	$\frac{18}{25}$	$-\frac{16}{25}$		

$-\frac{37}{4} - \frac{357}{100} = \frac{-925-357}{100} = -\frac{1276}{100} = -\frac{319}{25}$

$-\frac{1}{2}$	1	0	0	$\frac{1}{4}$	0	3
$\frac{1}{2}$	0	0	$\frac{4}{25}$	$\frac{7}{100}$	$-\frac{1}{25}$	$\frac{39}{25}$
0	1	0	$4/25$	$8/25$	$-1/25$	$4\frac{14}{25}$
$-\frac{5}{16}$	0	1	0	$-\frac{3}{32}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{5}{16}$	0	0	$\frac{1}{10}$	$\frac{7}{160}$	$-\frac{1}{40}$	$\frac{39}{40}$
0	0	1	$\frac{1}{10}$	$-\frac{1}{20}$	$\frac{1}{10}$	$\frac{11}{10}$

$\frac{3}{32} + \frac{7}{160}$

$= \frac{-15+7}{160} = \frac{-8}{160} = -\frac{1}{20}$

$\frac{1}{8} - \frac{1}{40} = \frac{5-1}{40} = \frac{4}{40} = \frac{1}{10}$

$\frac{1}{8} + \frac{39}{40} = \frac{5+39}{40} = \frac{44}{40} = \frac{11}{10}$

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio.
Z	0	0	0	$-9/25$	$-18/25$	$-4/25$	$-12\frac{19}{25}$	
x_1	1	0	0	$8/25$	$7/50$	$-2/25$	$78/25$	
x_2	0	1	0	$4/25$	$8/25$	$-1/25$	$4\frac{14}{25}$	
x_3	0	0	1	$11/10$	$-1/20$	$1/10$	$11/10$	

This is the optimal table and the optimal solution is given as:

$$Z = -12\frac{19}{25}, \quad x_1 = \frac{78}{25}$$

$$x_2 = 4\frac{14}{25}; \quad x_3 = \frac{11}{10}$$

Q:- Maximize:- Q_4

$$Z = 3x_1 + x_2 + 3x_3$$

Subject to

$$-x_1 + 2x_2 + x_3 \leq 4$$

$$4x_2 - 3x_3 \leq 2.$$

$$x_1 - 3x_2 + 2x_3 \leq 3$$

& $x_1, x_2, x_3 \geq 0$

Sol:-

The given problem in standard form is

$$Z = 3x_1 + x_2 + 3x_3$$

$$Z - 3x_1 - x_2 - 3x_3 = 0$$

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S/t

$$-x_1 + 2x_2 + x_3 + x_4 = 4$$
$$4x_2 - 3x_3 + x_5 = 2$$
$$x_1 - 3x_2 + 2x_3 + x_6 = 3$$

In tabular Form

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	-3	-1	-3	0	0	0	0	
x_4	-1	2	1	1	0	0	4	
x_5	0 P.E	4	-3	0	1	0	2	
← x_6	(1)	-3	2	0	0	1	3	$\frac{3}{1} = 3$
					-			

x_1 is entering and x_6 is leaving element where 1 is pivot element

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	-10	3	0	0	3	9	
x_4	0	-1	3	1	0	1	7	
← x_5	0	P.E (4)	-3	0	1	0	2	$\frac{2}{4} = \frac{1}{2}$
x_1	1	-3	2	0	0	1	3	

x_2 is entering and x_5 is leaving while 4 is pivot element

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	0	$-9/2$ P.E	0	10	3	14	
← x_4	0	0	(9/4)	1	1	1	15/2	$\frac{10}{3}$
x_2	0	1	$-3/4$	0	1	0	1/2	
x_1	1	0	$-1/4$	0	3	1	9/2	

x_3 is entering and x_4 is leaving while $\frac{9}{4}$ is a pivot element.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	0	0	0	2	12	5	29	
x_3	0	0	1	4/9	4/9	4/9	10/3	
x_2	0	1	0	1/3	4/3	1/3	3	
x_1	1	0	0	1/9	28/9	10/9	16/3	

This is the optimal table and the optimal solution is

$Z = 29$; $x_1 = \frac{16}{3}$

$x_2 = 3$; $x_3 = \frac{10}{3}$

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:Rough Work: Q4

$-3-\frac{3}{4}=\frac{-12-3}{4}$

-3	1	-3	0	0	0	0
0	1	-3/4	0	1	0	1/2
-3	0	-15/4	0	1	0	1/2
0	-10	3	0	0	3	9
0	10	-15/2	0	10	0	5
0	0	-9/2	0	10	3	14
0	-1	3	1	0	1	7
0	1	-3/4	0	1	0	1/2
0	0	9/4	1	1	1	15/2
1	-3	2	0	0	1	3
0	3	-9/4	0	3	0	3/2
1	0	-1/4	0	3	1	9/2
0	0	-9/2	0	10	3	14
0	0	9/2	2	2	2	15
0	0	0	2	12	5	29

Similarly remaining.

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Q:-5 Maximize :-

$$Z = 4x_1 + 6x_2 + 2x_3$$

Subject to

$$4x_1 - 4x_2 \leq 5$$
$$-x_1 + x_2 + x_3 \leq 5$$
$$-x_1 + 6x_2 \leq 5$$
$$x_1, x_2, x_3 \geq 0$$

Sol:-

The given problem is standard form is

$$Z = 4x_1 + 6x_2 + 2x_3$$

s.t

Z- Equation is

$$Z - 4x_1 - 6x_2 - 2x_3 = 0$$

s.t

$$4x_1 - 4x_2 + x_4 = 5$$
$$-x_1 + x_2 + x_3 + x_5 = 5$$
$$-x_1 + 6x_2 + x_6 = 5$$
$$x_i \geq 0, \quad i = 1, 2, \ldots, 6$$

In tabular Form :-

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	-4	-6	-2	0	0	0	0	
x_4	4	-4	0	1	0	0	5	$\frac{5}{4}$
x_5	-1	P.E 1	1	0	1	0	5	$\frac{5}{1}=5$
← x_6	-1	(6)	0	0	0	1	5	$\frac{5}{6}$

Here, x_2 is entering and x_6 is leaving while 6 is a pivot element.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	-5	0	-2	0	0	1	5	
	P.E							
← x_4	(10/3)	0	0	1	0	2/3	25/3	$\frac{25}{2}$
x_5	-5/6	0	1	0	1	-1/6	25/6	
x_2	-1/6	1	0	0	0	1/6	5/6	

Here;
x_1 is entering and x_4 is leaving while $\frac{10}{3}$ is pivot element.

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	0	0	-2	3/2	0	2	35/2	
x_1	1	0	0	3/10	0	1/5	5/2	
← x_5	0	0	(1) P.E	1/4	1	0	275/36	275/36
x_2	0	1	0	1/20	0	1/5	5/4	

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	0	0	2	2	2	$32\frac{7}{9}$	
x_1	1	0	0	3/10	0	1/5	5/2	
x_3	0	0	1	1/4	1	0	$7\frac{23}{36}$	
x_2	0	1	0	1/20	0	1/5	5/4	

This is the optimal table with optimal solution as:

$$Z = 32\frac{7}{9} \quad ; \quad x_1 = \frac{5}{2}$$

$$x_2 = \frac{5}{4} \quad ; \quad x_3 = 7\frac{23}{36}$$

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Rough Work Q5.

-4	-6	-2	0	0	0	0
-1	6	0	0	0	1	5
-5	0	-2	0	0	1	5
4	-4	0	1	0	0	5
$-2/3$	4	0	0	0	$2/3$	$10/3$
$10/3$	0	0	1	0	$2/3$	$25/3$
-1	1	1	0	1	0	5
$\mp 1/6$	± 1	± 0	± 0	± 0	$\pm 1/6$	$\pm 5/6$
$-5/6$	0	1	0	1	$-1/6$	$25/6$
-5	0	-2	0	0	1	5
5	0	0	$3/2$	0	1	$25/2$
0	0	-2	$3/2$	0	2	$35/2$
$-5/6$	0	1	0	1	$-1/6$	$25/6$
$5/6$	0	0	$1/4$	0	$1/6$	$125/36$
0	0	1	$1/4$	1	0	$\frac{275}{36}$
$-1/6$	1	0	0	0	$1/6$	$5/6$
$-1/6$	0	0	$1/20$	0	$1/30$	$5/12$
0	1	0	$1/20$	0	$1/5$	$5/4$

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Q:- Discuss the special cases in the simplex method.

Sol:-

There are the four special cases arises in the application of the simplex method.

i) Degeneracy.
ii) Alternative Optimum.
iii) Unbounded Solution.
iv) Infeasible Solution.

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i) Degeneracy:-

In the application of the feasible condition of the simplex method a tie for the minimum ratio may be broken arbitrarily for the purpose of determing of the leaving variable. When this happened one or more basic variable will be zero in the next alteration. In this case the new solution is degenerated. Degeneracy points out that there is at least one extra constraints in the modle.

Note:-

Unfortunately, there is not reliable method for the identification of extra constraints directly from the modle.

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Example:-

Consider, the problem.

Max.

$$Z = 3x_1 + 9x_2$$

Subject to

$$x_1 + 4x_2 \leq 8$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

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Sol:-

The given problem in standard form is

$$Z = 3x_1 + 9x_2$$

Z- Equation is

$$Z - 3x_1 - 9x_2 = 0$$

S.t

$$x_1 + 4x_2 + x_3 = 8$$
$$x_1 + 2x_2 + x_4 = 4$$

In tabular form.

Basic	x_1	x_2	x_3	x_4	Sol	Ratio
Z	-3	-9	0	0	0	
x_3	1	4	1	0	8	$\frac{8}{4} = 2$
← x_4	1	(2) P.E	0	1	4	$\frac{4}{2} = 2$

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Basic	x_1	x_2	x_3	x_4	Sol	Ratio.
Z	3/2	0	0	9/2	18	
x_3 ←	(-1) P.E	0	1	-2	0	
x_2	1/2	1	0	1/2	2	

This is the optimal table and optimal solution is

$Z = 18$; $x_2 = 2$, $x_1 = 0$

Let us enter x_1 and leave x_3.

Basic	x_1	x_2	x_3	x_4	Sol	Ratio
Z	0	0	3/2	3/2	18	
x_1	1	0	-1	2	0	
x_2	0	1	1/2	-1/2	2	

Sol. is

$Z = 18$

$x_1 = 0$

$x_2 = 2$

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ii) Alternative Optimum:

When the objective function parallel to the one of the constraints, then it is the indication that alternative optimum exists.

In simplex method if the coefficient of the non-basic variable in the objective equation of the solution is zero, this is an indication that ~~alter~~ alternative optimum exists.

Example:-

Consider the problem

Maximize.

$$Z = 2x_1 + 4x_2$$

Subject to

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Sol:- The given problem is standard form is

$$Z = 2x_1 + 4x_2$$

Z - Equation is

$$Z - 2x_1 - 4x_2 = 0$$

S/t

$$x_1 + 2x_2 + x_3 = 5$$

$$x_1 + x_2 + x_4 = 4$$

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In tabular form,

Basic	x_1	x_2	x_3	x_4	Sol	Ratio
Z	-2	-4	0	0	0	
		P.E				
← x_3	1	(2)	1	0	5	$\frac{5}{2}=2.5$
~~P.E~~						
x_4	1	(1)	0	1	4	$\frac{4}{1}=4$

↓

Basic	x_1	x_2	x_3	x_4	Sol	Ratio
Z	0	0	2	0	10	
	~~P.E~~					
x_2	(1/2)	1	1/2	0	5/2	
	P.E					
← x_4	(1/2)	0	~~-1/2~~	1	~~3~~/2	

This is the optimal table and the optimal solution is

$Z = 10$; $x_1 = 0$, $x_2 = 5/2$.

Let us observe that what happens if we further proceeds. Choose x_1 as entering and x_4 as leaving element. So;

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Basic	x_1	x_2	x_3	x_4	Sol.	Ratio
z	0	0	2	0	10	
x_2	1	1	1	-1	1	
x_1	1	0	-1	2	3	

This is also optimal table and optimal solution is

$$z = 10, \quad x_1' = 3, \quad x_2' = 1$$

Note:-

From these two solutions we obtain infinite number of solutions as follows.

$$\bar{x}_i = \lambda x_i + (1-\lambda) x_i' \qquad 0 \le \lambda \le 1$$
$$i = 1, 2$$

(iii) Unbounded Solution:

In some LP models the value of the variable increases infinitely without violating any of the constraints. It means that the solution space is unbounded in at least one direction. As a result the objective value may increase (maximization case) or decrease (minimization case) infinitely. In this case both the solution space and the optimum objective values are unbounded.

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(In simplex method if in any alteration the coefficients of a non-basic variable are all zero or -ive, then this is indication that the solution is unbounded.)

Example:-

Maximize

$$Z = 2x_1 + 3x_2$$

Subject to

$$x_1 - x_2 \leq 10$$
$$2x_1 \leq 40$$
$$x_1, x_2 \geq 0$$

Sol:-

The given problem in standard form is

$$Z = 2x_1 + 3x_2$$

Z - Equation is

$$Z - 2x_1 - 3x_2 = 0$$

S.t

$$x_1 - x_2 + x_3 = 10$$
$$2x_1 + x_4 = 40$$
$$x_1, x_2, x_3, x_4 \geq 0$$

In tabular form we have.

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Basic	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	-2	-1	0	0	0	
x_3	1	-1	1	0	10	
x_4	2	0	0	1	40	

(Since, x_2 is a non-basic variable and all the coefficients of x_2 are either zero or -ive. So, the solution is unbounded.)

Q:- Write a note On M-techniques:-

Sol:-

M-Technique OR
Big M-method OR
Method Of Penalty

If all the constraints of the origional problem are of less or equal type, then slack variables are used for starting solution.

Whoever, if the problem includes constraints of the type $\geq$, or, $=$, then the surplus or artifical variables are introduced to obtain a starting basic feasible solution. Artificial variables are usually denoted by $R_1, R_2, - - - - -$.

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In M-techniques after adding artifical variables in the constraints, we assign a very large per unit penality to these artifical variables in the objective function to insure that the artifical variable will be zero in the optimal solution.

In maximization problem, we penalize the artifical variables in the objective functions by "$-M$" and in the minimization problem by "$+M$". where $M \gg 0$.

In optimal table all the artifical variables must be zero. If at least one of the artifical variable is +ive in the optimal table, then this is indication that the problem has no feasible solution and the solution is called infeasible solution.

Q:- 6.# Solve the following problem by M-technique.

Minimize,

$$Z = 4x_1 + x_2$$

Subject to

$$3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4 \; ; \; x_1, x_2 \geq 0$$

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Sol:- The given problem in standard form is

minimize:

$$Z = 4x_1 + x_2$$

S/t

$$3x_1 + x_2 + R_1 = 3$$
$$4x_1 + 3x_2 - x_3 + R_2 = 6$$
$$x_1 + 2x_2 + x_4 = 4$$

So,

$$Z = 4x_1 + x_2 + MR_1 + MR_2 \quad , M \gg 0.$$

$$\Rightarrow Z = 4x_1 + x_2 + M(3 - 3x_1 - x_2) + M(6 - 4x_1 - 3x_2 + x_3)$$
$$\Rightarrow Z = 4x_1 + x_2 + 3M - 3Mx_1 - Mx_2 + 6M - 4Mx_1 - 3Mx_2 + Mx_3$$
$$Z = (4 - 7M)x_1 + (1 - 4M)x_2 + Mx_3 + 9M.$$

Z= Eq is:

$$\Rightarrow Z + (7M - 4)x_1 + (4M - 1)x_2 - Mx_3 = 9M$$

In Tabular Form:

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol	Ratio
Z	$7M-4$	$4M-1$	$-M$	0	0	0	$9M$	
← R_1	③ (P.E)	1	0	0	1	0	3	$\frac{3}{3}=1$
R_2	4	3	-1	0	0	1	6	$\frac{6}{4}=\frac{3}{2}$
x_4	1	2	0	1	0	0	4	$\frac{4}{1}=4$

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Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol	Ratio
Z	0	$\frac{5M+1}{3}$	$-M$	0	$\frac{1}{3}(4-7M)$	0	$2M+4$	
x_1	1	1/3	0	0	1/3	0	1	3
← R_2	0	5/3 (P.E)	-1	0	-4/3	1	2	$\frac{6}{5}$
x_4	0	5/3	0	1	-1/3	0	3	$\frac{9}{5}$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio.
Z	0	0	1/5	0	$\frac{8}{5}-M$	$-M-\frac{1}{5}$	18/5	
x_1	1	0	1/5	0	3/5	-1/5	3/5	$\frac{3/5}{1/5}=3$
x_2	0	1	-3/5	0	-4/5	3/5	6/5	
← x_4	0	0	1 (P.E)	1	1	-1	1	1

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol	Ratio.
Z	0	0	0	-1/5	$\frac{7}{5}-M$	$-M$	17/5	
x_1	1	0	0	-1/5	2/5	0	2/5	
x_2	0	1	0	3/5	-1/5	0	9/5	
x_3	0	0	1	1	1	-1	1	

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Since R_2 is a non-basic variable and all the coefficients of R_2 are either zero or -ive.

So, the solution space is unbounded.

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-:Rough Work Q6:-

4	3	-1	0	0	1	6
± 4	$\pm 4/3$	± 0	± 0	$\pm 4/3$	± 0	± 4
0	$5/3$	-1	0	$-4/3$	1	2

1	2	0	1	0	0	4
± 1	$\pm 1/3$	± 0	± 0	$\pm 1/3$	± 0	± 1
0	$5/3$	0	1	$-1/3$	0	3

$7M-4$	$4M-1$	$-M$	0	0	0	$9M$
$\pm 7M-4$	$\pm \frac{1}{3}(7M-4)$	± 0	± 0	$\pm \frac{1}{3}(7M-4)$	± 0	$\pm 7M-4$
0	$\frac{5M+1}{3}$	$-M$	0	$-\frac{1}{3}(7M-4)$	0	$2M+4$

1	$1/3$	0	0	$1/3$	0	1
± 0	$\pm 1/3$	$\mp 1/15$	± 0	$\mp 4/15$	$\pm 1/5$	$\pm 2/5$
1	0	$1/5$	0	$3/5$	$-1/5$	$3/5$

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$$\begin{matrix} 0 & 5/3 & 0 & 1 & -1/3 & 0 & 3 \\ \pm 0 & \pm 5/3 & \mp 1 & \pm 0 & \mp 4/3 & \pm 1 & \pm 2 \\ 0 & 0 & 1 & 1 & 1 & -1 & 1 \end{matrix}$$

$$\begin{matrix} 0 & \frac{5M+1}{3} & -M & 0 & \frac{4-7M}{3} & 0 & 2M+4 \\ \pm 0 & \pm\frac{5M+1}{3} & \mp\frac{3}{5}M\left(\frac{5M+1}{3}\right) & \pm 0 & \mp\frac{4}{5}\left(\frac{5M+1}{3}\right) & \mp\frac{2}{5}\left(\frac{5M+1}{3}\right) & \mp\frac{2}{5}\left(\frac{5M+1}{3}\right) \end{matrix}$$

$$0 \quad 0 \quad 1/5 \quad 0 \quad \frac{8}{5}-M \quad -\frac{1}{5}(5M+1) \quad 18/5$$

$$0 \quad 1 \quad -3/5 \quad 0 \quad -4/5 \quad 3/5 \quad 6/5$$

$$\begin{matrix} \pm 0 & \pm 0 & \mp 3/5 & \mp 3/5 & \mp 3/5 & \mp 3/5 & \mp 3/5 \\ 0 & 1 & 0 & 3/5 & -4/5 & 0 & 9/5 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 1/5 & 0 & 3/5 & -1/5 & 3/5 \\ \pm 0 & \pm 0 & \pm 1/5 & \mp 1/5 & \pm 1/5 & \mp 1/5 & \pm 1/5 \\ 1 & 0 & 0 & -1/5 & 2/5 & 0 & 2/5 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 1/5 & 0 & \frac{8}{5}-M & -M-\frac{1}{5} & \frac{18}{5} \\ \mp 0 & \pm 0 & \pm 1/5 & \mp 1/5 & \pm 1/5 & \mp 1/5 & \pm 1/5 \\ 0 & 0 & 0 & -1/5 & \frac{7}{5}-M & -M & 17/5 \end{matrix}$$

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Q:- 7.n Maximize

$$Z = 2x_1 + 3x_2 - 5x_3$$

Subject to

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

Sol:-

The given problem in standard form is

Max.

$$Z = 2x_1 + 3x_2 - 5x_3$$

S/t

$$x_1 + x_2 + x_3 + R_1 = 7$$
$$2x_1 - 5x_2 + x_3 - x_4 + R_2 = 10$$
$$x_1, x_2, x_3, x_4, R_1, R_2 \geq 0$$

Then,

$$Z = 2x_1 + 3x_2 - 5x_3 - MR_1 - MR_2 \rightarrow (*)$$

As;

$$x_1 + x_2 + x_3 + R_1 = 7$$
$$\Rightarrow R_1 = 7 - x_1 - x_2 - x_3$$

Also;

$$2x_1 - 5x_2 + x_3 - x_4 + R_2 = 10$$
$$\Rightarrow R_2 = 10 - 2x_1 + 5x_2 - x_3 + x_4$$

Equation (*) becomes:

$$Z = 2x_1 + 3x_2 - 5x_3 - M(7 - x_1 - x_2 - x_3) - M(10 - 2x_1 + 5x_2 - x_3 + x_4)$$
$$\Rightarrow Z = (2 + M + 2M)x_1 + (3 + M - 5M)x_2 + (-5 - M - M)x_3 - Mx_4 - 17M$$

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Z - Equation is

$$Z - (2+3M)x_1 - (3-4M)x_2 + (5-2M)x_3 + Mx_4 = -17M$$

In tabular Form

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
Z	$-(2+3M)$	$-(3-4M)$	$5-2M$	M	0	0	$-17M$	
R_1	1	1	1	0	1	0	7	$\frac{7}{1}=7$
← R_2	(2) P.E	−5	1	−1	0	1	10	$\frac{10}{2}=5$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
Z	0	$-(8+\frac{7}{2}M)$	$6-\frac{M}{2}$	$-(1+\frac{M}{2})$	0	$1+\frac{3}{2}M$	$10-2M$	
← R_1	0	(7/2) P.E	1/2	1/2	1	−1/2	2	$\frac{2}{7/2}=\frac{4}{7}$
x_1	1	−5/2	1/2	−1/2	0	1/2	5	

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
Z	0	0	50/7	1/7	$\frac{16}{7}+M$	$M-\frac{1}{7}$	162/7	
x_2	0	1	1/7	1/7	2/7	−1/7	4/7	
x_1	1	0	6/7	−1/7	5/7	1/7	45/7	

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This is the optimal table and the optimal solution is

$Z = \frac{102}{7}$; $x_1 = \frac{45}{7}$; $x_2 = \frac{4}{7}$

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:- Rough Work :- Q7

1	1	1	0	1	0	7
±1	∓5/2	±1/2	∓1/2	±0	±1/2	±5
0	7/2	1/2	1/2	1	-1/2	2

$-(2+3M)$	$4M-3$	$5-2M$	M	0	0	$-17M$
$(2+3M)$	$\frac{5}{2}(2+3M)$	$\frac{1}{2}(2+3M)$	$-\frac{1}{2}(2+3M)$	0	$\frac{1}{2}(2+3M)$	$5(2+3M)$
0	$-8-\frac{7}{2}M$	$6-\frac{M}{2}$	$-1-\frac{M}{2}$	0	$1+\frac{3}{2}M$	$10-2M$

1	-5/2	1/2	-1/2	0	1/2	5
±0	5/2	5/14	5/14	5/7	-5/14	10/7
1	0	6/7	-1/7	5/7	1/7	45/7

0	$-(8+\frac{7}{2}M)$	$6-\frac{M}{2}$	$-(1+\frac{M}{2})$	0	~~$1+\frac{3}{2}M$ $10-2M$~~
0	$(8+\frac{7}{2}M)$	$\frac{1}{7}(8+\frac{7}{2}M)$	$\frac{1}{7}(0+\frac{7}{2}M)$	$\frac{2}{7}(0+\frac{7}{2}M)$	
0	0	50/7	1/7	$\frac{16}{7}+M$	

$1+\frac{3}{2}M$	$10-2M$
$-\frac{1}{7}(0+\frac{7}{2}M)$	$\frac{4}{7}(0+\frac{7}{2}M)$
$M-1/7$	102/7

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Q:-8 Consider the problem.

Maximize.

$$Z = 2x_1 + 4x_2 + 4x_3 - 3x_4 \rightarrow (*)$$

Subject to

$$x_1 + x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Solve the problem using x_3 and x_4 as ~~basic~~ starting basic variables or starting feasible solution

Sol:-

Given,

$$x_1 + x_2 + x_3 = 4$$

$$\Rightarrow x_3 = 4 - x_1 - x_2$$

and

$$x_1 + 4x_2 + x_4 = 8$$

$$\Rightarrow x_4 = 8 - x_1 - 4x_2$$

Then Eq (*) becomes

$$Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

$$Z = 2x_1 + 4x_2 + 4(4 - x_1 - x_2) - 3(8 - x_1 - 4x_2)$$

$$Z = (2 - 4 + 3)x_1 + (4 - 4 + 12)x_2 + 16 - 24$$

Z - Eq. is

$$Z - x_1 - 12x_2 = -8$$

In tabular Form

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Basic	x_1	x_2	x_3	x_4	Sol	Ratio
Z	-1	-12	0	0	-8	
x_3	1	1	1	0	4	$\frac{4}{1}=4$
← x_4	1	(4) P.E	0	1	8	$\frac{8}{4}=2$

Basic	x_1	x_2	x_3	x_4	Sol	Ratio
Z	2	0	0	3	16	
x_3	3/4	0	1	-1/4	2	
x_2	1/4	1	0	1/4	2	

Since all the coefficients of non-basic variables are either +ive or zero. So, this is optimal table and the optimal solution is

$Z = 16, \quad x_1 = 0 \; ; \quad x_2 = 2$

Rough Work Q8.

1	1	1	0	4
±1/4	±1	±0	±1/4	±2
3/4	0	1	-1/4	2
-1	-12	0	0	-8
3	12	0	3	24
2	0	0	3	16

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Q:- Consider the problem.

Maximization

$$Z = x_1 + 2x_2 + x_3$$

Subject to

$$x_1 + x_2 + x_3 = 7$$

$$2x_1 - 5x_2 + x_3 \geq 10$$

$$x_1, x_2, x_3 \geq 0$$

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Sol:-

The standard form of the given problem is

Max.

$$Z = x_1 + 2x_2 + x_3$$

S|t

$$x_1 + x_2 + x_3 + R_1 = 7$$

$$2x_1 - 5x_2 + x_3 - x_4 + R_2 = 10$$

Then;

By M- Tech.

$$Z = x_1 + 2x_2 + x_3 - MR_1 - MR_2 \rightarrow (*) \quad (M \gg 0)$$

Sol:

$$R_1 = 7 - x_1 - x_2 - x_3$$

$$R_2 = 10 - 2x_1 + 5x_2 - x_3 + x_4$$

Eq (*) becomes

$$Z = x_1 + 2x_2 + x_3 - M(7 - x_1 - x_2 - x_3) - M(10 - 2x_1 + 5x_2 - x_3 + x_4)$$

$$= x_1 + 2x_2 + x_3 - 7M + Mx_1 + Mx_2 + Mx_3 - 10M + 2Mx_1 - 5Mx_2 + Mx_3 - Mx_4$$

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$Z = (1+M+2M)x_1 + (2+M-5M)x_2 + (1+M+M)x_3 - Mx_4 - (7M+10M)$.

$Z = (1+3M)x_1 + (2-4M)x_2 + (1+2M)x_3 - Mx_4 - 17M$.

Z - eq is.

$Z - (1+3M)x_1 + (4M-2)x_2 - (1+2M)x_3 + Mx_4 = -17M$.

In tabular Form.

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio.
Z	$-(1+3M)$	$(4M-2)$	$-(1+2M)$	M	0	0	$-17M$	
R_1	1	1	1	0	1	0	7	$\frac{7}{1}=7$
← R_2	(2) P.E	−5	1	−1	0	1	10	$\frac{10}{2}=5$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
Z	0	$-\frac{1}{2}(7M+9)$	$-\frac{1}{2}(1+M)$	$-\frac{1}{2}(1+M)$	0	$\frac{1}{2}(1+3M)$	$5-2M$	
← R_1	0	(7/2) P.E	1/2	1/2	1	−1/2	2	$\frac{1}{7}$
x_1	1	−5/2	1/2	−1/2	0	1/2	5	

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$\frac{1}{7}(7M+9)$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
Z	0	0	1/7	1/7	$\frac{1}{7}(7M+4)$	$\frac{1}{7}+M$	53/7	
x_2	0	1	1/7	1/7	2/7	-1/7	4/7	
x_1	1	0	6/7	-1/7	5/7	1/7	45/7	

This is the optimal table and the optimal sol is:

$Z = \frac{53}{7}$

$x_1 = \frac{45}{7}$

$x_2 = 4/7$

$x_3 = 0$

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-: Rough Work Q9 :-

1	1	1	0	1	0	7
±1	±5/2	±1/2	±1/2	±0	±1/2	±5
0	7/2	1/2	1/2	1	-1/2	2

$-(1+3M)$	$(4M-2)$	$-(1+2M)$	M	0	0	$-17M$
$(1+3M)$	$-\frac{5}{2}(1+3M)$	$\frac{1}{2}(1+3M)$	$-\frac{1}{2}(1+3M)$	0	$\frac{1}{2}(1+3M)$	$5(1+3M)$
0	$-\frac{7}{2}M-\frac{9}{2}$	$-\frac{1}{2}(1+M)$	$-\frac{1}{2}(1+M)$	0	$\frac{1}{2}(1+3M)$	$5-2M$

1	-5/2	1/2	-1/2	0	1/2	5
0	5/2	5/14	5/14	5/7	-5/14	10/7
1	0	6/7	-1/7	5/7	1/7	45/7

0	$-\frac{1}{2}(7M+9)$	$-\frac{1}{2}(1+M)$	$-\frac{1}{2}(1+M)$
0	$\frac{1}{2}(7M+9)$	$\frac{1}{14}(7M+9)$	$\frac{1}{14}(7M+9)$
0	0	1/7	1/7

0	$\frac{1}{2}(1+3M)$	$5-2M$
$\frac{1}{7}(7M+9)$	$-\frac{1}{14}(7M+9)$	$\frac{2}{7}(7M+9)$
$\frac{1}{7}(7M+9)$	$\frac{1}{7}+M$	53/7

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Q 10-11 Consider the problem.

Max.

$$Z = 3x_1 + 2x_2.$$

Subject to.

$$2x_1 + x_2 \leq 2.$$
$$3x_1 - 4x_2 \geq 12.$$
$$x_1, x_2 \geq 0.$$

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Sol:-

The problem in standard form is.

Max.

$$Z = 3x_1 + 2x_2$$

S.t

$$2x_1 + x_2 + x_3 = 2.$$
$$3x_1 + 4x_2 - x_4 + R = 12.$$
$$x_1, x_2, x_3, x_4, R \geq 0.$$

Then:

$$Z = 3x_1 + 2x_2 - MR. \rightarrow (*)$$

As:

$$3x_1 + 4x_2 - x_4 + R = 12.$$
$$\Rightarrow R = 12 - 3x_1 - 4x_2 + x_4.$$

Eq (*) becomes.

$$Z = 3x_1 + 2x_2 - M(12 - 3x_1 - 4x_2 + x_4)$$
$$Z = 3x_1 + 2x_2 - 12M + 3Mx_1 + 4Mx_2 - Mx_4$$
$$Z = (3+3M)x_1 + (2+4M)x_2 - Mx_4 - 12M$$

Z - Eq is

$$Z - (3+3M)x_1 - (2+4M)x_2 + Mx_4 = -12M.$$

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In Tabular Form:-

Basic	x_1	x_2	x_3	x_4	R	Sol.	Ratio
Z	$-(5+3M)$	$-(2+4M)$	0	M	0	$-12M$	
← x_3	2	(1) P.E	1	0	0	2	$\frac{2}{1}=2$
R	3	4	0	-1	1	12	$\frac{12}{4}=3$

Basic	x_1	x_2	x_3	x_4	R	Sol.	Ratio
Z	~~$-17+11M$~~	0	$(2+4M)$	M	0	$4(1-M)$	
x_2	2	1	1	0	0	2	
R	-5	0	-4	-1	1	4	

This is the optimal table and since R the artifical variable is 4 (i.e $R \neq 0$).

∴ The given problem has no feasible solution.

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Rough Work Q10..

3	4	0	-1	1	12
±8	±4	±4	±0	±0	±8
-5	0	-4	-1	1	4
-(3+3M)	-(2+4M)	0	M	0	-12M
2(2+4M)	(2+4M)	(2+4M)	0	0	2(2+4M)
~~-(3+3M)~~ 1+5M	0	(2+4M)	M	0	4(1-M)

Q:- Write a note on Two phase Technique.

Two Phase Technique:-

Possible A deficiency of the M-technique is the ~~feasible~~ computational error, that may results from assigning a very large value to constant "M".

The two phase method is designed to remove ~~alleviate~~ this deficiency. By eliminating the constant "M" altogether the

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artifical variable added to the same as in M-technique, however the artifical variables are not panalized in the objective function. As the name suggested that this method solve the LP problems in two phase.

The algorithm for this method is as follows.

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Phase I:-

Put the problem in LP standard form and add the necessary artifical variables to obtain a starting basic solution. Construct a new objective function that seeks the minimization of the sum of the artifical variables subjected to the constraints of the origional ~~function~~ problem. If the minimum value of the new objective function is +ive, then the LP problem has no feasible solution. If the minimum value of the new objective function is zero, then the problem has a feasible solution. Then,

We move to phase II.

Phase II:-

Eliminate from the optimal

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solution of phase I, the column corresponding to the artifical variables and replace the objective ~~function~~ equation with objective equation of the origional problem to obtain a starting solution of the origional problem and proceed further to find optimal solution of the origional problem.

Q1: Solve by two phase method.

Min.

$$Z = 4x_1 + x_2$$

s/t

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

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Sol:-

Given problem in standard form.

Min.

$$Z = 4x_1 + x_2$$

s/t

$$3x_1 + x_2 + R_1 = 3$$
$$4x_1 + 3x_2 - x_3 + R_2 = 6$$
$$x_1 + 2x_2 + x_4 = 4$$

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Phase I:-

Min.

$$W = R_1 + R_2 \rightarrow (*)$$

As;

$$R_1 = 3 - 3x_1 - x_2$$

$$R_2 = 6 - 4x_1 - 3x_2 + x_3$$

Eq (*) becomes.

$$W = 3 - 3x_1 - x_2 + 6 - 4x_1 - 3x_2 + x_3$$

$$W = -7x_1 - 4x_2 + x_3 + 9$$

$\Rightarrow$ W- Eq becomes.

$$W + 7x_1 + 4x_2 - x_3 = 9$$

In Tabular Form:-

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
W	7	4	-1	0	0	0	9	
	P.E							
← R_1	(3)	1	0	0	1	0	3	$\frac{3}{3} = 1$
R_2	4	3	-1	0	0	1	6	$\frac{6}{4} = \frac{3}{2}$
x_4	1	2	0	1	0	0	4	$\frac{4}{1} = 4$

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In Tabular Form

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
W	0	5/3	-1	0	-7/3	0	2	
x_1	1	1/3	0	0	1/3	0	1	3
← R_2	0	(5/3) P.E	-1	0	-4/3	1	2	6/5
x_4	0	5/3	0	1	-1/3	0	3	9/5

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
W	0	0	0	0	-1	-1	0 ✓	
x_1	1	0	1/5	0	3/5	-1/5	3/5	
x_2	0	1	-3/5	0	-4/5	3/5	6/5	
x_4	0	0	1	1	1	-1	1	

Since, Min $W = 0$

So, we move to phase II.

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Phase :- II

Min

$$Z = 4x_1 + x_2.$$

s/t

$$x_1 + \frac{1}{5}x_3 = \frac{3}{5} \Rightarrow x_1 = \frac{3}{5} - \frac{1}{5}x_3$$

$$x_2 - \frac{3}{5}x_3 = \frac{6}{5} \Rightarrow x_2 = \frac{6}{5} + \frac{3}{5}x_3$$

$$x_3 + x_4 = 1$$

Putting in Z.

$$Z = 4\left(\frac{3}{5} - \frac{1}{5}x_3\right) + \left(\frac{6}{5} + \frac{3}{5}x_3\right).$$

$$Z = \frac{12}{5} - \frac{4}{5}x_3 + \frac{6}{5} + \frac{3}{5}x_3$$

$$Z = \frac{18}{5} - \frac{1}{5}x_3$$

Z - Eq is:

$$Z + \frac{1}{5}x_3 = \frac{18}{5}$$

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In Tabular form we have

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Rough Work Q1.a

7	4	-1	0	0	0	9
±7	±7/3	±0	±0	±7/3	±0	±7
0	5/3	-1	0	-7/3	0	2.

4	3	-1	0	0	1	6
±4	±4/3	±0	±0	±4/3	±0	±4
0	5/3	-1	0	-4/3	1	2.

1	2	0	1	0	0	4
±1	±1/3	±0	±0	±1/3	±0	±4/3
0	5/3	0	1	-1/3	0	3

0	5/3	-1	0	-7/3	0	2
±0	±5/3	∓1	±0	∓4/3	±1	±2
0	0	0	0	-1	-1	0

1	1/3	0	0	1/3	0	1
±0	±1/3	∓1/5	±0	∓4/15	±1/5	±2/5
1	0	1/5	0	3/5	-1/5	3/5

0	0	1/5	0	18/5
±0	±0	∓1/5	±1/5	±1/5
0	0	0	-1/5	17/5

1	0	1/5	0	3/5
±0	±0	±1/5	±1/5	±1/5
1	0	0	-1/5	2/5

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Basic	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	0	1/5	0	18/5	
x_1	1	0	1/5	0	3/5	$\frac{3/5}{1/5} = 3$
x_2	0	1	-3/5	0	6/5	
← x_4	0	0	(1) P.E	1	1	$\frac{1}{1} = 1$

Basic	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	0	0	-1/5	17/5	
x_1	1	0	0	-1/5	2/5	
x_2	0	1	0	3/5	9/5	
x_3	0	0	1	1	1	

This is the optimal table and the optimal solution is:

$Z = \frac{17}{5}$, $x_1 = \frac{2}{5}$, $x_2 = \frac{9}{5}$, $x_3 = 1$, $x_4 = 0$.

0 1 -3/5 0 6/5
0 0 3/5 3/5 3/5
0 1 0 3/5 9/5

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Q2. Consider the problem.

Max.

$$Z = 2x_1 + 3x_2 - 5x_3$$

Subject to.

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0.$$

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Sol:-

The given problem in standard form is.

$$Z = 2x_1 + 3x_2 - 5x_3$$

S|t

$$x_1 + x_2 + x_3 + R_1 = 7$$
$$2x_1 - 5x_2 + x_3 - x_4 + R_2 = 10.$$
$$x_1, x_2, x_3, x_4, R_1, R_2 \geq 0.$$

Phase I:-

Min.

$$W = R_1 + R_2 \rightarrow (*)$$

As;

$$R_1 = 7 - x_1 - x_2 - x_3$$
$$R_2 = 10 - 2x_1 + 5x_2 - x_3 + x_4$$

Eq (*) becomes.

$$W = 7 - x_1 - x_2 - x_3 + 10 - 2x_1 + 5x_2 - x_3 + x_4$$
$$W = -3x_1 + 4x_2 - 2x_3 + x_4 + 17.$$

W - Eq becomes.

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$W + 3x_1 - 4x_2 + 2x_3 - x_4 = 17$

In Tabular form:-

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
W	3	-4	2	-1	0	0	17	
R_1	1	1	1	0	1	0	7	$\frac{7}{1}=7$
← R_2	(2) P.E	-5	1	-1	0	1	10	$\frac{10}{2}=5$

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol	Ratio
W	0	7/2	1/2	1/2	0	-3/2	2	
← R_1	0	(7/2) P.E	1/2	1/2	1	-1/2	2	$\frac{4}{7}$
x_1	1	-5/2	1/2	-1/2	0	1/2	5	

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol	Ratio
W	0	0	0	0	-1	-1	0	
x_2	0	1	1/7	1/7	2/7	-1/7	4/7	
x_1	1	0	6/7	-1/7	5/7	1/7	45/7	

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:Rough Work Qd:-

3	-4	2	-1	0	0	17
±3	∓15/2	±3/2	∓3/2	±0	±3/2	±15
0	7/2	1/2	1/2	0	-3/2	2

0	7/2	1/2	1/2	0	-3/2	2
±0	±7/2	±1/2	±1/2	±1	∓1/2	±2
0	0	0	0	-1	-1	0

1	-5/2	1/2	-1/2	0	1/2	5
±0	∓5/2	±5/14	∓5/14	∓5/7	∓5/14	∓10/7
1	0	6/7	-1/7	5/7	1/7	45/7

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Since,

Min $W = 0$.

So, we move to phase II.

Phase II:-

Max.

$$Z = 2x_1 + 3x_2 - 5x_3.$$

S/t

$$x_2 + \frac{1}{7}x_3 + \frac{1}{7}x_4 = 4/7$$

$$x_1 + \frac{6}{7}x_3 - \frac{1}{7}x_4 = \frac{45}{7}.$$

So Then;

$$\Rightarrow x_2 = \frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}x_4.$$

$$\Rightarrow x_1 = \frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}x_4.$$

Then;

$$Z = 2\left(\frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}x_4\right) + 3\left(\frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}x_4\right) - 5x_3$$

$$Z = \frac{90}{7} - \frac{12}{7}x_3 + \frac{2}{7}x_4 + \frac{12}{7} - \frac{3}{7}x_3 - \frac{3}{7}x_4 - 5x_3$$

$$Z = -\frac{50}{7}x_3 - \frac{1}{7}x_4 + \frac{102}{7}$$

$$\Rightarrow Z + \frac{50}{7}x_3 + \frac{1}{7}x_4 = \frac{102}{7}.$$

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In Tabular Form:-

Basic	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	0	50/7	1/7	102/7	.
x_2	0	1	1/7	1/7	4/7	
x_1	1	0	6/7	-1/7	45/7	

This is the optimal Table and optimal solution is.

$$Z = \frac{102}{7} \; ; \quad x_1 = \frac{45}{7}$$

$$x_2 = \frac{4}{7} \; ; \quad x_3 = 0 \; , \quad x_4 = 0$$

Q3.11 Consider the problem.

Max. $Z = x_1 + 2x_2 + x_3$.

Subject to.

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

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Sol:- The given problem in standard form is

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Max.

$$Z = x_1 + 2x_2 + x_3$$

s/t

$$x_1 + x_2 + x_3 + R_1 = 7$$

$$2x_1 - 5x_2 + x_3 - x_4 + R_2 = 10$$

Phase I:- Minimum

$$W = R_1 + R_2$$

$$W = 7 - x_1 - x_2 - x_3 + 10 - 2x_1 + 5x_2 - x_3 + x_4$$

$$W = -3x_1 + 4x_2 - 2x_3 + x_4 + 17$$

$$\Rightarrow W + 3x_1 - 4x_2 + 2x_3 - x_4 = 17$$

In tabular form.

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol	Ratio
W	3	-4	2	-1	0	0	17	
R_1	1	1	1	0	1	0	7	$\frac{7}{1}=7$
← R_2	(2) P.E	-5	1	-1	0	1	10	$\frac{10}{2}=5$

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Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol	Ratio
W	0	7/2	1/2	1/2	0	-3/2	2	
← R_1	0	(7/2) P.E	1/2	1/2	1	-1/2	2	4/7
x_1	1	-5/2	1/2	-1/2	0	1/2	5	

Basic	x_1	x_2	x_3	x_4	R_1	R_2	Sol.	Ratio
W	0	0	0	0	-1	-1	0	
x_2	0	1	1/7	1/7	2/7	-1/7	4/7	
x_1	1	0	6/7	-1/7	5/7	1/7	45/7	

This is the optimal table and minimum

$W = 0$

So, we move to phase II.

Phase II:-

Max.

$$Z = x_1 + 2x_2 + x_3$$

S|t

$$x_2 + \frac{1}{7}x_3 + \frac{1}{7}x_4 = 4/7$$

$$x_1 + \frac{6}{7}x_3 - \frac{1}{7}x_4 = 45/7$$

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$$\Rightarrow x_2 = \frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}x_4.$$

$$\Rightarrow x_1 = \frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}x_4.$$

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Z - Eq becomes.

$$Z = \left(\frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}x_4\right) + 2\left(\frac{4}{7} - \frac{1}{7}x_3 - \frac{1}{7}x_4\right) + x_3$$

$$Z = \frac{45}{7} - \frac{6}{7}x_3 + \frac{1}{7}x_4 + \frac{8}{7} - \frac{2}{7}x_3 - \frac{2}{7}x_4 + x_3$$

$$Z = \frac{53}{7} - \frac{1}{7}x_3 - \frac{1}{7}x_4$$

$$\Rightarrow Z + \frac{1}{7}x_3 + \frac{1}{7}x_4 = \frac{53}{7}$$

Basic	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	0	1/7	1/7	53/7	
x_2	0	1	1/7	1/7	4/7	
x_1	1	0	6/7	-1/7	45/7	

This is the optimal table and the optimal solution is:

$Z = 53/7$; $x_1 = 45/7$

$x_2 = 4/7$; $x_3 = 0$; $x_4 = 0$.

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Rough Work Q3.

3	-4	2	-1	0	0	17
÷3	÷15/2	÷3/2	÷3/2	÷0	÷3/2	÷15/2
0	7/2	1/2	1/2	0	-3/2	2
1	1	1	0	1	0	7
÷1	÷5/2	÷1/2	÷1/2	÷0	÷1/2	÷5
0	7/2	1/2	1/2	1	-1/2	2
0	7/2	1/2	1/2	0	-3/2	2
0	÷7/2	÷1/2	÷1/2	÷1	÷1/2	÷2
0	0	0	0	-1	-1	0
1	-5/2	1/2	-1/2	0	1/2	5
0	5/2	5/14	5/14	5/7	-5/14	10/7
1	0	6/7	-1/7	5/7	1/7	45/7

INFEASIBILITY:-

If all the constraints in LP model can't be solved simultaneously, then the model has no feasible solution.

If we solve the problem by M-method and the solution of at least one artifical variable is non-zero, then the problem has infeasible solution. Also in case of two phase method if the minimum value of the

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objective function in P-I is non-zero, then, this is indication that the solution is infeasible.

Q:- Consider the problem.

Maximize.

$$Z = 3x_1 + 2x_2$$

Subject to

$$2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

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Sol:-

We solve the problem by Big-M method.

See Previous Q10:-

Q2:- Using the Two phase method. Show that the problem has infeasible solution.

Max.

$$Z = 3x_1 + 2x_2$$

S.t

$$2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0.$$

Sol:-

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The given problem in standard form is:

Max.

$$Z = 3x_1 + 2x_2$$

s/t

$$2x_1 + x_2 + x_3 = 2$$
$$3x_1 + 4x_2 - x_4 + R = 12$$

Phase I:-

Minimum.

$$W = R$$

s/t

$$2x_1 + x_2 + x_3 = 2$$
$$3x_1 + 4x_2 - x_4 + R = 12$$
$$\Rightarrow R = 12 - 3x_1 - 4x_2 + x_4$$
$$x_1, x_2, x_3, x_4, R \geq 0$$

Then:

$$W = 12 - 3x_1 - 4x_2 + x_4$$
$$\Rightarrow W + 3x_1 + 4x_2 - x_4 = 12$$

In Tabular Form:-

Basic	x_1	x_2 ↓	x_3	x_4	R	Sol.	Ratio
W	3	4	0	-1	0	12	
← x_3	2	(1) P.E	1	0	0	2	$\frac{2}{1} = 2$
R	3	4	0	-1	1	12	$\frac{12}{4} = 3$

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Basic	x_1	x_2	x_3	x_4	R	Sol	Ratio
W	-5	0	-4	-1	0	4	
x_2	2	1	1	0	0	2	
R	-5	0	-4	-1	1	4	

Since all the coefficients of non-basic variable in W-row are zero or -ive So, this is optimal table. and min $W = 4 \neq 0$.

∴ The problem has no feasible solution.

Q:- Discuss the difference between M-tech. and two phase method.

Sol:-

M-Technique	Two Phase Method
(i) M-technique uses the modified objective function with penality for artifical variable.	(i) Two-phase method uses two objective function, the origional objective function and the other which is sum of all artifical variables.
(ii) M-technique solve the problem in one stage. (step).	(ii) Two phase method solve the problem in two stages (steps).

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(iii) M-tech is not very suitable for computer implementation.	(iii) Two phase method is better suitable for computer implementation.
iv) M-tech may leads to incorrect solution because of error due to very large value of M.	iv) No such problem arises in two phase method.

Q:- Write a note on dual Simplex Method.

Dual Simplex Method:- (DSM)

This method starts better than optimal but infeasible solution and moves to achieve feasibility while maintaining optimality.

To solve the problem by DSM we proceeds as follows.

i) Select the leaving variable by using the feasibility condition.

Feasibility Condition:-

The leaving variable is a variable having the most -ive value in the solution coulumn.

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(ii) Select the entering variable by using the optimality conditions.

Optimality Conditions:-

Take the ratio of the L.H.S coefficients of the Z-equation to the corresponding coefficients in the equation associated with the leaving variable. Ignore the ratio associated with zero or +ive denominator.

(iii) The entering variable associated with smallest absolute ratio.

iv) Select the pivot element and use the Gauss Jordan method to ~~obtain~~ obtain a new alteration.

v) Repeat the process until the feasible solution is achieved.

Q 1.0 Solve the following problem by dual Simplex Method.

Min

$$Z = 2x_1 + x_2$$

Subject to

$$3x_1 + x_2 \geq 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

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Sol:-

The given problem can be written as.

Min.

$Z = 2x_1 + x_2$, Z-Eq is $Z - 2x_1 - x_2 = 0$.

S/t

$$-3x_1 - x_2 \leq -3$$
$$-4x_1 - 3x_2 \leq -6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$

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In standard form.

$$Z - 2x_1 - x_2 = 0$$

S/t

$$-3x_1 - x_2 + x_3 = -3$$
$$-4x_1 - 3x_2 + x_4 = -6$$
$$x_1 + 2x_2 + x_5 = 3.$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

In Tabular form.

Basic	x_1	x_2	x_3	x_4	x_5
Z					
x_3					
x_4					
x_5					

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Basic	x_1	x_2	x_3	x_4	x_5	Sol.
Z	-2	-1	0	0	0	0
x_3	-3	-1	1	0	0	-3
← x_4	-4	-3 (P.E)	0	1	0	-6
x_5	1	2	0	0	1	3
Ratio	$\left\|\frac{-2}{-4}\right\| = \frac{1}{2}$	$\left\|\frac{-1}{-3}\right\| = \frac{1}{3}$				

Basic	x_1	x_2	x_3	x_4	x_5	Sol.
Z	$-2/3$	0	0	$-1/3$	0	2
← x_3	$-5/3$ (P.E)	0	1	$-1/3$	0	-1
x_2	$-4/3$	1	0	$-1/3$	0	2
x_5	$-5/3$	0	0	$2/3$	1	-1
Ratio	$\left\|\frac{-2/3}{-5/3}\right\| = \frac{2}{5}$			$\left\|\frac{-1/3}{-1/3}\right\| = 1$		

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Basic	x_1	x_2	x_3	x_4	x_5	Sol.
Z	0	0	-2/5	-1/5	0	12/5
x_1	1	0	-3/5	1/5	0	3/5
x_2	0	1	4/5	-3/5	0	6/5
x_5	0	0	-1	1	1	0
Ratio.						

This is the optimal table and the optimal solution is.

$$Z = \frac{12}{5} \quad ; \quad x_1 = \frac{3}{5}$$

$$x_2 = \frac{6}{5} \quad ; \quad x_3 = 0.$$

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-: Rough Work Q1:-

-2	-1	0	0	0	0
4/3	1	0	-1/3	0	2
-2/3	0	0	-1/3	0	2

-3	-1	1	0	0	-3
4/3	1	0	-1/3	0	2
-5/3	0	1	-1/3	0	-1

1	2	0	0	1	3
±8/3	±2	0	∓2/3	0	±4
-5/3	0	0	2/3	1	-1

-2/3	0	0	-1/3	0	2
2/3	0	-2/5	2/15	0	2/5
0	0	-2/5	-1/5	0	12/5

4/3	1	0	-1/3	0	2
±4/3	±0	∓4/5	±4/15	0	±4/5
0	1	4/5	-3/5	0	6/5

-5/3	0	0	2/3	1	-1
5/3	0	-1	1/3	0	1
0	0	-1	1	1	0

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Q2.4 Solve the following problem by DSM.

Minimum.

$$Z = 2x_1 + 3x_2$$

Subject to.

$$2x_1 + 3x_2 \leq 30$$
$$x_1 + 2x_2 \geq 10$$
$$x_1, x_2 \geq 0.$$

Sol:-

The given problem can be written as.

Minimum.

$$Z = 2x_1 + 3x_2$$

Z - Eq is

$$Z - 2x_1 - 3x_2 = 0$$

S/t

$$2x_1 + 3x_2 \leq 30$$
$$-x_1 - 2x_2 \leq -10$$

In standard form.

$$Z - 2x_1 - 3x_2 = 0$$

s/t.

$$2x_1 + 3x_2 + x_3 = 30$$
$$-x_1 - 2x_2 + x_4 = -10$$

In tabular Form.

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Basic	x_1	x_2	x_3	x_4	Sol
z	-2	-3	0	0	0
x_3	2	3	1	0	30
← x_4	-1	(P.E) -2	0	1 ⊗	-10
Ratio.	$\left\|\frac{-2}{-1}\right\| = 2$	$\left\|\frac{-3}{-2}\right\| = \frac{3}{2}$			

Basic	x_1	x_2	x_3	x_4	Sol ~~Ratio.~~
z	$-1/2$	0	0	$-3/2$	15
x_3	$1/2$	0	1	$3/2$	15
x_2	$1/2$	1	0	$-1/2$	5
Ratio.					

This is the optimal table and optimal solution is $z = 15$, $x_1 = 15$; $x_2 = 5$

:: Rough Work :- Q2.

$-2 \quad -3 \quad 0 \quad 0 \quad 0$

$3/2 \quad 3 \quad 0 \quad -3/2 \quad 15$

$-1/2 \quad 0 \quad 0 \quad -3/2 \quad 15$

$2 \quad 3 \quad 1 \quad 0 \quad 30$

$\pm 3/2 \quad \pm 3 \quad \pm 0 \quad \pm -3/2 \quad \pm 15$

$1/2 \quad 0 \quad 1 \quad 3/2 \quad 15$

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Assignment:-

Consider the following

Minimum.

$$Z = 5x_1 + 6x_2.$$

Subject to

$$x_1 + x_2 \geq 2.$$
$$4x_1 + x_2 \geq 4$$
$$x_1, x_2 \geq 0$$

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Sol:-

Given problem can be written as.

$$Z = 5x_1 + 6x_2.$$

Z - Eq. is

$$Z - 5x_1 - 6x_2 = 0$$

S/t

$$-x_1 - x_2 \leq -2$$
$$-4x_1 - x_2 \leq -4$$

In standard form we have.

$$Z - 5x_1 - 6x_2 = 0.$$

S/t

$$-x_1 - x_2 + x_3 = -2.$$
$$-4x_1 - x_2 + x_4 = -4$$

In tabular form we have

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Basic	x_1	x_2	x_3	x_4	Sol.
Z	-5	-6	0	0	0
x_3	-1	-1	1	0	-2
← x_4	(-4) P.E	-1	0	1	-4.
Ratio.	$\left\|\frac{-5}{-4}\right\| = \frac{5}{4}$	$\left\|\frac{-6}{-1}\right\| = 6$			

Basic	x_1	x_2	x_3	x_4	Sol.
Z	0	-19/4	0	-5/4	5
← x_3	0	(-3/4) P.E	1	-1/4	-1
x_1	1	1/4	0	-1/4	1
Ratio.		$\left\|\frac{-19/4}{-3/4}\right\| = \frac{19}{3}$			

Basic	x_1	x_2	x_3	x_4	Sol.
Z	0	0	-19/3	1/3	34/3
x_2	0	1	-4/3	1/3	4/3
x_1	1	0	1/3	-4/3	2/3.
Ratio.					

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:- Rough Work:- Assignment

-5	-6	0	0	0
5	5/4	0	-5/4	5
0	-19/4	0	-5/4	5
-1	-1	1	0	-2
1	1/4	0	-1/4	1
0	-3/4	1	-1/4	-1
0	-19/4	0	-5/4	5
0	19/4	-19/3	19/12	19/3
0	0	-19/3	1/3	34/3
1	1/4	0	-1/4	1
±0	± 1/4	∓1/3	± 1/12	± 1/3
1	0	1/3	-4/3	2/3

This is the optimal table and optimal solution is

$Z = 34/3$; $x_1 = 2/3$

$x_2 = 4/3$

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Duality:-

The LP model we develop for a solution is refferred as primal problem. Each LP problem has a second problem associated with it. One problem is called primal problem and other is called dual. The dual problem is constructed from the primal expressed in standard form as follows.

i) If the primal seeks maximization then the dual seeks minimization and vice versa.

ii) Each constraint in the primal corresponds to a variable in the dual.

iii) The R.H.S ↑ elements of the constraints in the primal are the coefficients of the respective variables in the objective function of the dual.

iv) Each variable in the primal corresponds to a constraint in the dual.

v) The constraints coefficients of the primal variables form the L.H.S coefficients of the corresponding dual constraints.

vi) If the dual seeks maximization then its constraints are of form "≤"

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and if dual seeks minimization then the constraints will be of the form "≥".

vii) The objective coefficient of a primal variable becomes the R.H.S of the corresponding dual constraint.

Q1.) Write the dual of the following problem.

Maximize

$$Z = 5x_1 + 12x_2 + 4x_3$$

Subject to

$$x_1 + 2x_2 + x_3 \leq 10$$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1, x_2, x_3 \geq 0$$

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Sol:-

Primal can be written in standard form as:

Max

$$Z = 5x_1 + 12x_2 + 4x_3$$

s.t

$$x_1 + 2x_2 + x_3 + x_4 = 10$$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

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Dual is :-

Min.

$$W = 10y_1 + 8y_2$$

Subject to.

$$y_1 + 2y_2 \geq 5$$
$$2y_1 - y_2 \geq 12$$
$$y_1 + 3y_2 \geq 4$$
$$y_1 \geq 0$$

where;

y_2 is unrestricted.

Note :-

To make an unrestricted variable say y_2 to be restricted.

Putting

$$y_2 = y_2' - y_2''$$

Q2.11 Write the dual of the following problem

Min.

$$Z = 15x_1 + 12x_2.$$

Subject to

$$x_1 + 2x_2 \geq 3.$$
$$2x_1 - 4x_2 \leq 5$$

Sol:-

Given problem can be written as.

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Min.

$Z = 15x_1 + 12x_2$.

S.t

~~$x_1 + 2x_2 \geq 3$~~. $x_1 + 2x_2 - x_3 = 3$.

~~$2x_1 - 4x_2 \leq 5$~~ $2x_1 - 4x_2 + x_4 = 5$

~~$x_1, x_2 \geq 0$~~. $x_1, x_2, x_3, x_4 \geq 0$

Dual is

Max.

$W = 3y_1 + 5y_2$

Subject to

$y_1 + 2y_2 \leq 15$

$2y_1 - 4y_2 \leq 12$.

$-y_3 \leq 0 \Rightarrow y_1 \geq 0$

$y_2 \leq 0$.

Also ; $-y_2 \geq 0 \Rightarrow y_2' \geq 0$ $(y_2' = -y_2)$

Impi:-

Primal Dual Relationship:-

The primal and the dual problem are so closely related that the optimal solution of one problem can be written directly ~~f~~ from the optimal simplex ~~problem~~ table of the other problem.

This result is based on the

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following properties.

i) At any simplex alteration of the primal or dual

{Object Coefficients of variable j in one problem} = {L.H.S - R.H.S of constraints j in other problem.}

This property is symmetrical w.r.t both the primal and the dual.

For any pair of feasible primal and dual solution.

{Objective Value in the maximization problem} ≤ {Objective value in minimization problem}.

At the optimum the relation holds as stricked equation.

Q1.

Max

$$Z = x_1 + 2x_2 + 3x_3$$

Subject to

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 - x_2 = 4$$
$$x_1, x_2, x_3 \geq 0$$

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Using the starting solution consisting of x_3 and an artifical variable are in the 2nd constraint we obtain the following optimal table.

B	x_1	x_2	x_3	R	Sol.
Z	0	2	0	$-1+M$	5
x_1	0	5/2	1	$-1/2$	1
x_2	1	$-1/2$	0	1/2	2

Write the dual of above problem and find its solution from the optimal table of the primal

Sol:-

Primal in standard form can be written as.

Max.

$$Z = x_1 + 2x_2 + 3x_3 - MR$$

S.t

$$x_1 + 2x_2 - x_3 = 3$$
$$2x_1 - x_2 + R = 4$$
$$x_1, x_2, x_3, R \geq 0$$

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Dual is :-

Min.

$$W = 3y_1 + 4y_2$$

S.t

$$y_1 + 2y_2 \geq 1$$
$$2y_1 - y_2 \geq 2$$
$$y_1 \geq 3$$
$$y_2 \geq -M$$

By the primal dual relationship for the starting basic variable. x_3 and R.

We have.

For Variable x_3 ←

$$y_1 - 3 = 0$$
$$\Rightarrow y_1 = 3$$

For Variable R ←

$$y_2 - (-M) = -1 + M$$
$$\Rightarrow y_2 + M = -1 + M$$
$$\Rightarrow y_2 = -1$$

Putting y_1 and y_2 in "W" we get

$$W = 3(3) + (4)(-1)$$
$$W = 9 - 4$$
$$W = 0.5$$

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(9)

So, the solution of dual is

$W = 5$

$y_1 = 3$

$y_2 = -1$

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Q2.11 Maximize.

$$Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

Subject to:

$$x_1 + x_2 + x_3 = 4$$

$$x_1 + 4x_2 + x_4 = 8.$$

Using x_3 and x_4 as starting variable the optimal table is given by

B	x_1	x_2	x_3	x_4	Sol.
Z	0	0	0	3	16
x_3	3/4	0	1	-1/4	2
x_4	1/4	1	0	1/4	2

Write the dual problem and its solution by using above table.

Sol:-

The given primal in standard form is:

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(8)

Max.

$$Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

s/t

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$

Dual is :-

Min.

$$W = 4y_1 + 8y_2.$$

s/t

$$y_1 + y_2 \geq 2.$$
$$y_1 + 4y_2 \geq 4$$
$$y_1 \geq 4$$
$$y_2 \geq -3.$$

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Now;

By primal dual relationship for x_3 and x_4 we have.

For Variable x_3 &

$$y_1 - 4 = 0$$
$$\Rightarrow y_1 = 4$$

For Variable x_4 &

$$y_2 + 3 = 3$$
$$\Rightarrow y_2 = 0.$$

Putting in "W" we get

$$W = 4(4) + 8(0)$$
$$W = 16$$

∴ Solution of dual is.

$W = 16$; & $y_1 = 4$, $y_2 = 0$.

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82.

Conversion Of Real Life Problems In Mathematical Model And Graphical Method:-

Q1. Power Plus is an electrical appliance manufacturing company in Islamabad and hence is ~~is~~ producing three kinds of electrical appliances which requires four different process. The main power requirements are.

Man's hour Spent.

Products.	Metal-Work	Fitting	Electrical	Finishing
Refrigerator	0.2	~~0.5~~	0.5	0.3	0.3
Washing Machine	0.4	~~0.6~~	0.6	0.6	0.1
A.C	0.2	~~0.5~~	0.5	0.5	0.2.

In any one day three are 60 men hour available in metal work section, 125 men available in fitting section, 132 men hour available in electrical section and 30 men hour in finishing section. Power Plus gol : Rs. 700/- profit on each ~~refriger~~ refrigerator, Rs- 800/- on each washing machine and Rs- 400/- on each A.C.

(93)

Formulate the problem as LP model for maximizing profit.

Sol:- Let,

x_1 = No. of refrigrator.
x_2 = No. of washing machine.
x_3 = A.C's.

are produced in each day.

Then,

mathematical model of this problem is.

Max.

$$Z = 700x_1 + 800x_2 + 400x_3.$$

Subject to

$$0.2x_1 + 0.4x_2 + 0.2x_3 \leq 60$$
$$0.5x_1 + 0.6x_2 + 0.5x_3 \leq 125$$
$$0.3x_1 + 0.6x_2 + 0.5x_3 \leq 132$$
$$0.3x_1 + 0.1x_2 + 0.2x_3 \leq 30.$$
$$x_1, x_2, x_3 \geq 0.$$

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Q2. An auto workshop needs 650 cans of paints per month, the supply consists of 3 different clours.

Black Rs. 160 per can.
Blue Rs. 180 per can
Green Rs. 170 per can.

The painting works requires al least 100 cans of black paint; not more than 175 cans of blue paint and at least 65 cans of green paints.

How many cans of each of three paints should be purchased to minimize the total cost.

Sol:

Let,

x_1 = No. of cans of black paint
x_2 = No. of cans of blue paint
x_3 = No. of cans of green paint purchased per month

Then the mathematical ~~problem~~ model of this problem is.

Minimize:

$$Z = 160x_1 + 180x_2 + 170x_3$$

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(Q5)

Subject to.

$x_1 + x_2 + x_3 = 650$

$x_1 \geq 100$

$x_2 \leq 175$

$x_3 \geq 65$

$x_1, x_2, x_3 \geq 0$

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Q3. A manufacturing company produces two products A and B which are processed through departments X, Y and Z, and the number of hours require to finish each product is indicated in the following table.

Departments	Products A	Products B	Max. hour available per month
X	7	8	224
Y	8	12	192
Z	15	16	480

If the profit for the products of Rs-6 for a unit of product A and

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(96)

Rs. 4 for a unit of product B. What quantities per month should be produced to maximize profit. Formulate the problem as LP model.

Sol:- Let;

x_1 = No. of units of products A

x_2 = No. of unit of product B. produced in one month.

~~Max.~~ The mathematical model of this problem is

Max.

$$Z = 6x_1 + 4x_2.$$

Subject to.

$$7x_1 + 8x_2 \leq 224$$
$$8x_1 + 12x_2 \leq 192$$
$$15x_1 + 16x_2 \leq 480.$$
$$x_1, x_2 \geq 0.$$

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(87)

Q4. An airways flies 3 kinds of ~~aeroplanes~~ aeroplanes of long range, short range and medium range. The airline requires the aeroplanes on lease from a manufacturer. The annual profit contribution of the plane is long range 7, medium range 5 and of short range is 3 million. The airlines employers 15 engineers, 30 pilots and 80 airhostesses for the smooth running to business. Each long rang plane requires 3 engineers, for ~~maintenance~~ maintenance, 4 pilot and 6 airhostesses. Each medium range plane requires 5 engineers, 3 pilots and 5 airhostess and each short range plane requires 4 engineers, 2 pilot and 7 airhostesses. The airliner objective is to determine the number of the planes of each type to maximize the profit. Formulate the problem as LP model.

Sol:-

Let;

x_1 = No. of the long range aeroplanes.

x_2 = No. of the medium range aeroplane.

x_3 = No. of the short range aeroplane.

acquire from the manufacturer.

The,

the mathematical model of this

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problem is:

Max.

$$Z = 7x_1 + 5x_2 + 3x_3.$$

Subject to.

$$3x_1 + 5x_2 + 4x_3 \leq 15$$
$$4x_1 + 3x_2 + 2x_3 \leq 30$$
$$6x_1 + 5x_2 + 7x_3 \leq 80$$
$$x_1, x_2, x_3 \geq 0.$$

Q6.# Assignment:-

A company manufacture two types of screw wooden and metal. Each screw has to pass through two machines. slotting and threading. Each machine can work for 42 hours per week. 100 wooden screw requires 10 minutes on threading and 15 minutes on the slotting machine. 100 metal screw requires 25 minutes and 18 minutes for threading and slotting respectively. The profit is Rs. 50 per hundrad for wooden screws and Rs. 72 per hundrad for metal screws.

Formulate the problem as an LP model to maximize the profit.

Sol:-

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(99) ایسے سوال جن میں Variables 2 ہوں اور سوال Graphical Method سے حل ہوگا۔

-:. GRAPHICAL METHOD:-

Question:- 1.

Minimize the following problem by the graphical method.

Minimize

$$Z = 3x_1 + 2x_2.$$

Subject to

$$5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12.$$
$$x_1 + 4x_2 \geq 12.$$
$$x_1, x_2 \geq 0.$$

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Sol:-

Step I:

The given problem can be written as

Min.

$$Z = 3x_1 + 2x_2.$$

S|t

$$5x_1 + x_2 = 10.$$
$$\Rightarrow \frac{1}{2}x_1 + \frac{1}{10}x_2 = 1 \quad (l_1) \rightarrow (1)$$

$$2x_1 + 2x_2 = 12.$$
$$\Rightarrow \frac{1}{6}x_1 + \frac{1}{6}x_2 = 1 \quad (l_2) \rightarrow (2)$$

$$x_1 + 4x_2 = 12.$$
$$\Rightarrow \frac{1}{12}x_1 + \frac{1}{3}x_2 = 1 \quad (l_3) \rightarrow (3)$$

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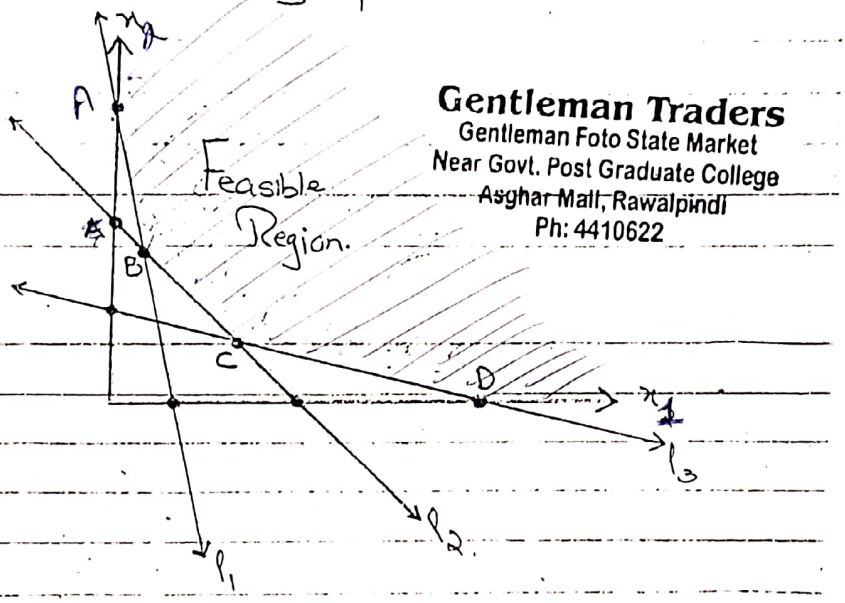
(100)

Step II :

Sketch the graph of constraint lines.



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Here, A, B, C and D are corner points of feasible region.

From fig. we have.

A(0, 10) , D(12, 0).

For B, we solve l_2 and l_1

Xing l_1 by 2 and subtracting l_1 and l_2 we get

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(101)

$$10x_1 + 2x_2 = 20$$
$$\pm 2x_1 \pm 2x_2 = \pm 12$$

$$8x_1 + 0 = 8.$$

$$\Rightarrow x_1 = 1.$$

Then,

$$5x_1 + x_2 = 10$$
$$5(1) + x_2 = 10$$
$$x_2 = 10 - 5$$

$$\Rightarrow x_2 = 5$$

So,

$$B(1, 5).$$

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For C, we solve l_2 and l_3.

Xing l_2 by 2 and subtracting l_2 and l_3 we get.

$$4x_1 + 4x_2 = 24$$
$$\pm x_1 \pm 4x_2 = \pm 12$$

$$3x_1 + 0 = 12$$

$$\Rightarrow x_1 = 4.$$

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(102)

As;

$$x_1 + 4x_2 = 12.$$
$$4 + 4x_2 = 12.$$
$$4x_2 = 12 - 4$$
$$x_2 = \frac{8}{4}$$
$$x_2 = 2$$

So,

~~D(4, 2) C(4, 2).~~

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~~Hence;~~

~~$Z = 3x_1 +$~~

Step III

As;

$$Z = 3x_1 + 2x_2.$$

Using $A(0, 10)$.

$$\Rightarrow Z = 20$$

Using $B(1, 5)$.

$$\Rightarrow Z = 13.$$

Using $C(4, 2)$.

$$\Rightarrow Z = 16.$$

Using $D(12, 0)$

$$\Rightarrow Z = 36.$$

Hence;

$Z = 13$
$x_1 = 1$
$x_2 = 5$

Ans:- (کیونکہ ہمیں Minimize کرنا ہے لہذا Z کو minimum قیمت ہی Sol. ہو گی۔)

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(10)

Q2:- Consider the following problem and solve by Graphical Method.

Max.

$$Z = 7x_1 + 5x_2.$$

S.T

$$x_1 + 2x_2 \leq 6.$$
$$4x_1 + 3x_2 \leq 12.$$
$$x_1, x_2 \geq 0.$$

Sol:-

Step I

Convert the constraints inequalities into equality by we have the following problem.

Max.

$$Z = 7x_1 + 5x_2.$$

S.T

$$x_1 + 2x_2 = 6. \rightarrow (i)$$
$$4x_1 + 3x_2 = 12. \rightarrow (ii)$$
$$x_1, x_2 \geq 0.$$

$$\Rightarrow \frac{1}{6}x_1 + \frac{1}{3}x_2 = 1 \rightarrow (1)\ l_1.$$

$$\Rightarrow \frac{1}{3}x_1 + \frac{1}{4}x_2 = 1 \rightarrow (2)\ l_2.$$

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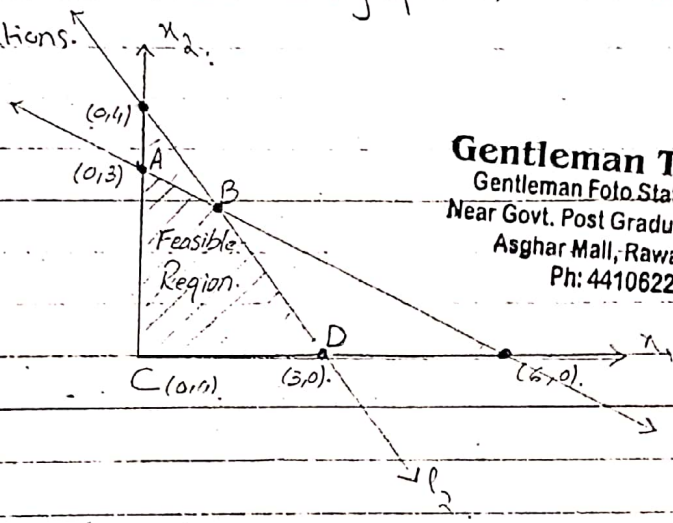
(04)

Step II :

Sketch the graph of the following equations.



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ABCD is a feasible region where, A, B C and D are corner points.

From fig:-

A(0,3), C(0,0), D(3,0).

For B:

We solve l_1 and l_2

Xing Eq (i) by 4 and subtracting (i) and (ii)

$$4x_1 + 8x_2 = 24$$
$$\pm 4x_1 \pm 3x_2 = \pm 12.$$

$$5x_2 = 12$$

$$\Rightarrow x_2 = \frac{12}{5}$$

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(15)

(i) ⇒

$$x_1 + 2\left(\frac{12}{5}\right) = 6.$$

$$\therefore x_1 = 6 - \frac{24}{5}$$

$$x_1 = \frac{30-24}{5}$$

$$x_1 = \frac{6}{5}$$

So; $B\left(\frac{6}{5}, \frac{12}{5}\right)$.

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Step III :-

As.

$$Z = 7x_1 + 5x_2.$$

Using $A(0,3)$

$$\Rightarrow Z = 15$$

Using $B\left(\frac{6}{5}, \frac{12}{5}\right)$

$$\Rightarrow Z = \frac{102}{5}$$

Using $C(0,0)$.

$$\Rightarrow Z = 0$$

Using $D(3,0)$.

$$\Rightarrow Z = 21$$

∴ Optimal solution is

$\{ Z = 21 ; \quad x_1 = 3, \quad x_2 = 0 \}$ Ans.

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(106)

Q:- Solve by Graphical Method.

Min.

$$Z = 2x_1 + 3x_2.$$

Subject to

$$2x_1 + x_2 \geq 8$$
$$-x_1 + x_2 \geq 6.$$
$$x_1 + 2x_2 \geq 9$$
$$x_1, x_2 \geq 0.$$

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Sol:-

Step I:-

Convert the constraints into the equalities.

Min.

$$Z = 2x_1 + 3x_2$$

S/t

$$2x_1 + x_2 = 8.$$

$$\Rightarrow \frac{1}{4}x_1 + \frac{1}{8}x_2 = 1 \longrightarrow (1)\ l_1.$$

$$x_1 + x_2 = 6$$

$$\Rightarrow \frac{1}{6}x_1 + \frac{1}{6}x_2 = 1 \longrightarrow (2)\ l_2.$$

$$x_1 + 2x_2 = 9$$

$$\Rightarrow \frac{1}{9}x_1 + \frac{1}{9/2}x_2 = 1 \longrightarrow (3)\ l_3.$$

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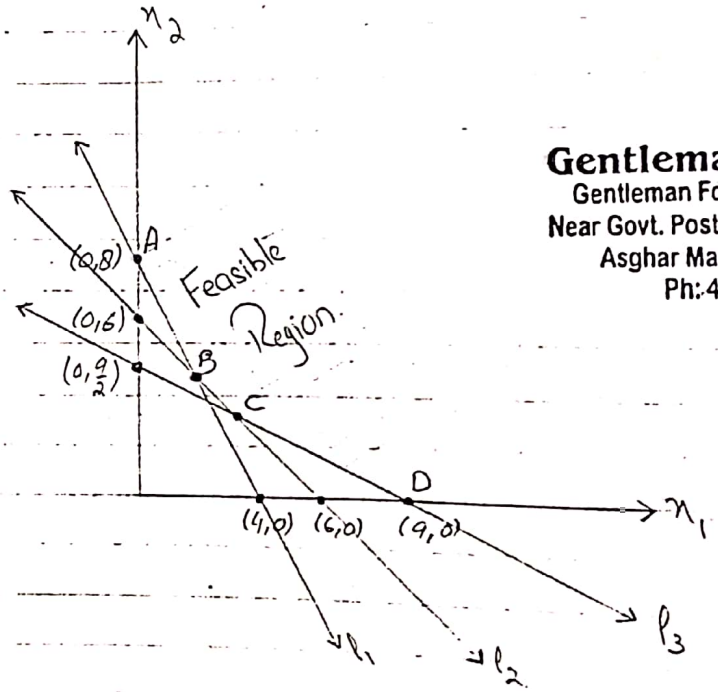
(157)

Step II :-

Sketch the graph.



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Corner points are A, B, C and D of feasible region.

$\therefore$ A (0, 8) , D (9, 0)

For Point B :

Solving l_1 and l_2 we have.

Subtracting Eq ① and ②.

$$2x_1 + x_2 = 8$$
$$\pm x_1 \pm x_2 = \pm 6$$
$$x_1 = 2.$$

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(188)

Eq ① $\Rightarrow$ $2(2) + x_2 = 8$

$\Rightarrow x_2 = 4.$

So;

$B(2, 4).$

For Point C

Solving l_2 and l_3.

Subtracting l_2 and l_3 we get

$$x_1 + x_2 = 6$$
$$\pm x_1 \pm 2x_2 = \pm 9$$
$$-x_2 = -3.$$
$$x_2 = 3.$$

Eq ② $\Rightarrow$ $x_1 + 3 = 6$

$\Rightarrow x_1 = 3.$

So;

$C(3, 3).$

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Step III

Ans.

$Z = 2x_1 + 3x_2$

Using $A(0, 8)$

$\Rightarrow Z = 24$

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(108)

Using B(2, 4)

$\Rightarrow Z = 16$

Using C(3, 3).

$\Rightarrow Z = 15$

Using D(9, 0).

$\Rightarrow Z = 18.$

So;

Optimal Sol. is

$\Rightarrow Z = 15, x_1 = 9, x_2 = 0$ } Ans:-

Note:-

In maximization problem, we have to restrict ourself under the line ground towards the left.

In minimization problem we will go beyond the line ground toward the right ~~as~~ according to inequalities.

Question:-4.11

Minimize.

$Z = 2x_1 + x_2$

Subject to

$x_1 + x_2 \geq 1$

$x_1 - x_2 \geq -1$

$x_1 + 2x_2 \geq 4$

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$x_1, x_2 \geq 0$

Sol:-

Step I :-

Convert the constraints into equality.

Min.

$Z = 2x_1 + x_2$

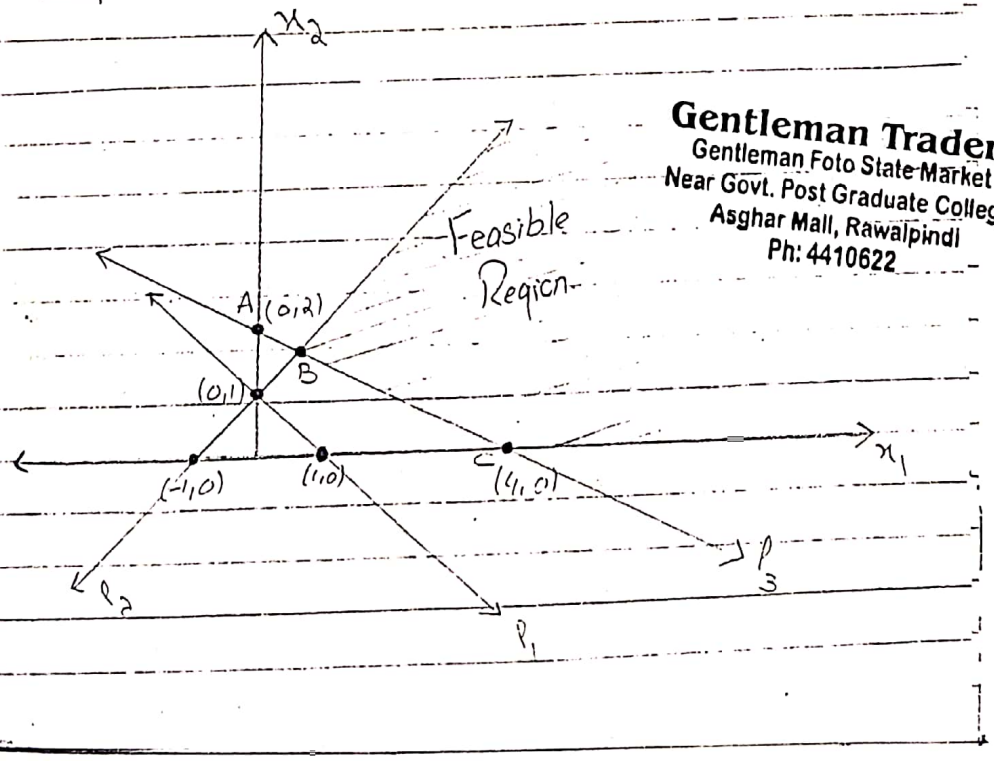
s/t

$x_1 + x_2 = 1 \rightarrow$ ① l_1.

$x_1 - x_2 = -1 \Rightarrow -x_1 + x_2 = 1 \rightarrow$ ② l_2.

$x_1 + 2x_2 = 4 \Rightarrow \frac{1}{4}x_1 + \frac{1}{2}x_2 = 1 \rightarrow$ ③ l_3.

Step II :-



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(iii)

Corner points of feasible solution are B and C. where.

~~A(0,1)~~, ~~C(1,0)~~. C(4,0).

For Point B.

Solving l_2 and l_3.

Substracting Eq ② and ③.

$$x_1 - x_2 = -1$$
$$\pm x_1 \pm 2x_2 = \pm 4$$
$$-3x_2 = -5$$
$$x_2 = \frac{5}{3}$$

Eq ② ⇒

$$x_1 - x_2 = -1$$
$$x_1 - \frac{5}{3} = -1$$
$$x_1 = \frac{5}{3} - 1$$
$$x_1 = \frac{2}{3}$$

So,

$$B\left(\frac{2}{3}, \frac{5}{3}\right).$$

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(112)

Step III

As,

$Z = 2x_1 + x_2$

Using ~~A(0,2)~~ C(4,0).

~~$\Rightarrow Z = 2$~~

$\Rightarrow Z = 8$.

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Using $B\left(\frac{2}{3}, \frac{5}{3}\right)$.

$\Rightarrow Z = 2\left(\frac{2}{3}\right) + \left(\frac{5}{3}\right)$

$= \frac{4+5}{3} = \frac{9}{3}$

$\Rightarrow Z = 3$.

Optimal Sol. is:

$Z = 3, \quad x_1 = \frac{2}{3}, \quad x_2 = \frac{5}{3}$ } Ans:-

Assignment:-

Consider the problem. Maximize.

~~Minimize~~

$Z = 500x_1 + 300x_2$

Subject to.

$x_1 + 2x_2 \leq 16$

$2x_1 + x_2 \leq 11$

$3x_1 + x_2 \leq 15$

$x_1, x_2 \geq 0$

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(11.3)

Sol:-

Step I :-

Convert the constraints into equalities.

So, Maximize.

$$Z = 500x_1 + 300x_2.$$

S.t

$$x_1 + 2x_2 = 16.$$

$$\Rightarrow \frac{1}{16}x_1 + \frac{1}{8}x_2 = 1 \longrightarrow (1)\ l_1$$

$$2x_1 + x_2 = 11.$$

$$\Rightarrow \frac{1}{11/2}x_1 + \frac{1}{11}x_2 = 1 \longrightarrow (2)\ l_2.$$

$$3x_1 + x_2 = 15$$

$$\Rightarrow \frac{1}{5}x_1 + \frac{1}{15}x_2 = 1 \longrightarrow (3)\ l_3.$$

Step II :-

Draw the graph of l_1, l_2 and l_3.

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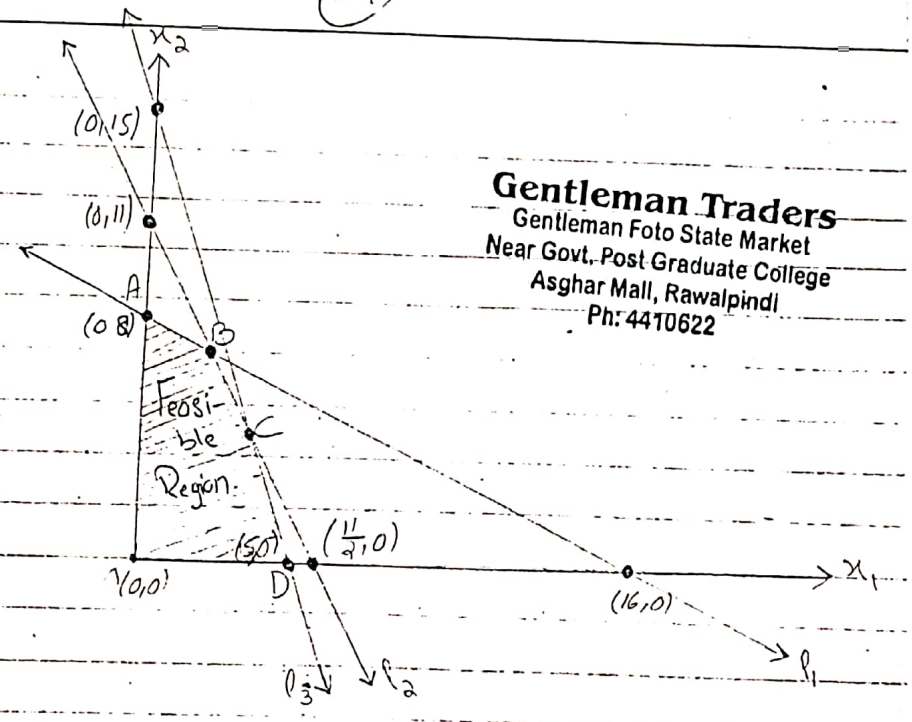
(Ex 4)



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Corner points of feasible region are O, A, B, C and D.

Here;

O(0,0), A(0,8), C = ?, B = ? D(5,0).

For Point B:

Solving l_2 and l_3 we get.

Xing Eq ① by ② and subtracting l_1 and l_2.

$$2x_1 + 4x_2 = 32$$
$$\pm 2x_1 \pm x_2 = \pm 11$$
$$3x_2 = 21$$
$$\Rightarrow x_2 = 7$$

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(iii)

from Eq (1):

$x_1 + 2(7) = 16$

$\Rightarrow x_1 = 16 - 14$

$\Rightarrow x_1 = 2.$

So:

$B(2, 7)$

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For Point C:

We solve l_2 and l_3.

Subtracting Eq (2) and Eq (3):

$2x_1 + x_2 = 11$

$\pm 3x_1 \pm x_2 = 15$

$-x_1 = -4$

$\Rightarrow x_1 = 4.$

From Eq (3):

$2(4) + x_2 = 11$

$\Rightarrow x_2 = 3.$

So:

$C(4, 3)$

Step III:

At:

$Z = 500x_1 + 300x_2$

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(16)

Using O(0,0).
$\Rightarrow Z = 0$

Using A(0, 8)
$\Rightarrow Z = 2400$

Using B(2, 7)
$\Rightarrow Z = 3100$

Using C(4, 3).
$\Rightarrow Z = 2900$

Using D(5, 0).
$\Rightarrow Z = 2500$

So,
Optimal sol. is:
$Z = 3100$
$x_1 = 2 \;;\; x_2 = 7$ } Ans:-

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Assignment:-

Consider the following problem.

Min
$Z = 5x + 5y.$

s/t
$5x + 4y \geq 40.$
$x + 2y \geq 14.$
$x + 3y \geq 18$
$x, y \geq 0$

Sol:-

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Step I:

Convert the constraint into equalities.

Min.

$$Z = 5x + 5y.$$

Subject to

$$5x + 4y = 40 \Rightarrow \frac{1}{8}x + \frac{1}{10}y = 1 \rightarrow (1)\ l_1$$

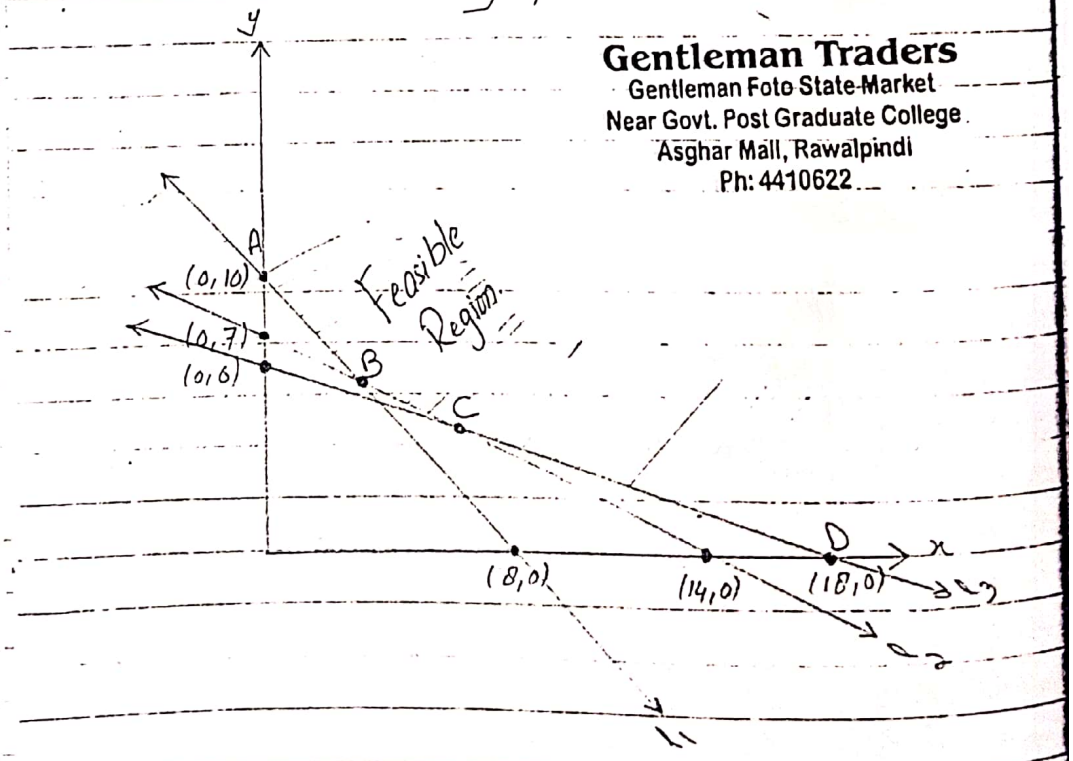
$$x + 2y = 14 \Rightarrow \frac{1}{14}x + \frac{1}{7}y = 1 \rightarrow (2)\ l_2$$

$$x + 3y = 18 \Rightarrow \frac{1}{18}x + \frac{1}{6}y = 1 \rightarrow (3)\ l_3$$

$$x, y \geq 0.$$

Step II:

Draw the graph



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(C8)

A, B, C and D are the corner points of a feasible region.

Here:

A (0, 10), D (18, 0), B = ?, C = ?

For Point B:

Solving l_1 and l_2

Xing Eq ② by ② and subtracting ① & ②

$$5x + 4y = 40$$
$$\pm 2x \pm 4y = \pm 28$$
$$3x = 12$$
$$x = \frac{12}{3}$$
$$\Rightarrow x = 4$$

From Eq ①

$$5(4) + 4y = 40$$
$$\Rightarrow 4y = 40 - 20$$
$$\Rightarrow 4y = 20$$
$$\Rightarrow y = 5$$

So:

B (4, 5)

For C Point:

Solving l_2 and l_3 we get

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(113)

Subtracting Eq ② and ③

$$x + 2y = 14$$
$$\pm x \pm 3y = \pm 18$$
$$-y = -4$$
$$\Rightarrow y = 4.$$

From Eq ②

$$x + 2(4) = 14$$
$$x = 14 - 8$$
$$x = 6.$$

So,

$$C(6, 4).$$

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Step III

As,

$$Z = 5x + 5y$$

Using $A(0, 10)$.

$$\Rightarrow Z = 50$$

Using $B(4, 5)$.

$$\Rightarrow Z = 45$$

Using $C(6, 4)$.

$$\Rightarrow Z = 50.$$

Using $D(18, 0)$

$$\Rightarrow Z = 90$$

So,

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Optimal sol. is:
$Z = \cancel{40}\ 45$; $x_1 = \cancel{10}\ 4$, $y = \cancel{6}\ 5$ } Ans:-

Q:-5. Suppose that a manufacturer is making two product x and y with the following requirements.

Products	Raw Material	Machine Time	Labour Time.
x	4 Tons	2 hr.	1 hr.
y	1 Tons	1 hr	1 hr.

The inputs are available in the following form per week of 90 tons. of raw material, 50 hrs. of machine time and 40 hrs. of labour time.

The manufacturer gets Rs. 400 on each unit of product x and Rs. 300 on each unit of product y.

Read the problem carefully and write the mathematical model for maximizing the profit. Also solve the problem by Graphical method.

Sol:-

Let;

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(12)

x_1 = No. of units of product x.
x_2 = " " " " " y.
are produced per week.

Then,
the mathematical model of the problem is.

Max.
$$Z = 400x_1 + 300x_2.$$

Subject to.
$$4x_1 + x_2 \leq 90$$
$$2x_1 + x_2 \leq 50$$
$$x_1 + x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

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Now, we solve it by Graphical Method.

Step I:
Convert the constraints into equality.

Max.
$$Z = 400x_1 + 300x_2$$

s/t
$$4x_1 + x_2 = 90$$
$$\Rightarrow \frac{1}{45/2}x_1 + \frac{1}{90}x_2 = 1 \longrightarrow (1)\ l_1$$

$$2x_1 + x_2 = 50$$
$$\Rightarrow \frac{1}{25}x_1 + \frac{1}{50}x_2 = 1 \longrightarrow (2)\ l_2$$

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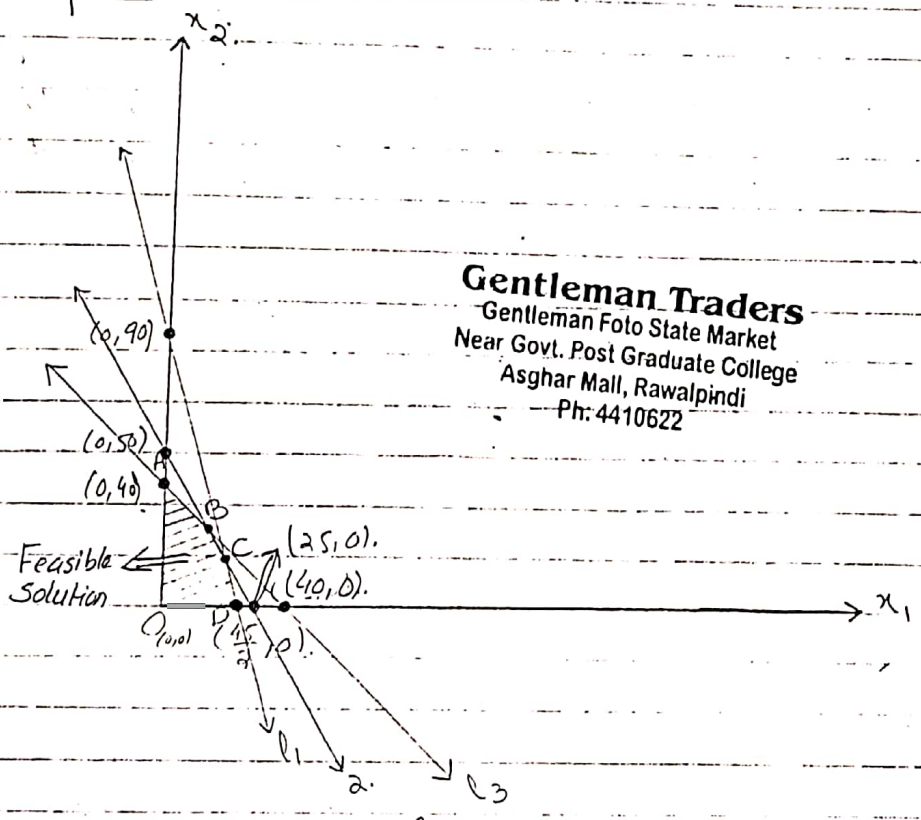
(122)

$x_1 + x_2 = 40$

$\Rightarrow \frac{1}{40}x_1 + \frac{1}{40}x_2 = 1 \longrightarrow (3)\ l_3$.

Step II



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Corner points of feasible solution are O, A, B, C and D.

Here;

O(0,0), A(0,40), B = ?, C = ? D$(\frac{45}{2},0)$.

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(123)

For B point

Solving l_2 and l_3.

Subtracting Eq ② and ③

$$2x_1 + x_2 = 50$$
$$\pm x_1 \pm x_2 = \pm 40$$

$$x_1 = 10.$$

From Eq ②:

$$2(10) + x_2 = 50.$$
$$\Rightarrow x_2 = 30.$$

So:

$$B(10, 30).$$

For Point C

Solving l_1 and l_2

Subtracting Eq ① and ②

$$4x_1 + x_2 = 90$$
$$\pm 2x_1 \pm x_2 = \pm 50$$

$$2x_1 = 40$$

$$\Rightarrow x_1 = 20$$

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(24)

Eq (1) becomes

$4(20) + x_2 = 90$

$\Rightarrow x_2 = 10$

So;

$C(20, 10)$.

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Step III &

As;

$Z = 400x_1 + 300x_2$.

Using $O(0,0)$

$\Rightarrow Z = 0$

Using $A(0, 40)$.

$\Rightarrow Z = 12000$

Using $B(10, 30)$.

$\Rightarrow Z = 13000$.

Using $C(20, 10)$.

$\Rightarrow Z = 11000$.

Using $D(\frac{45}{2}, 0)$.

$\Rightarrow Z = 9000$.

Optimal solution is

$Z = 13000, \quad x_1 = 10, \quad x_2 = 30$ } Ans:-

300
40
12000

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(125)

2007

Q.6 Three products are process through different operations. The time (minutes) required per unit of each product, the daily capacity of operations (in minutes/day) and the profit per unit sold of each product (in Rs) are as follows.

Operations	Product 1	Product 2	Product 3	Operation. Capacity.
1	3	0	2	430
2	1	4	0	460
3	3	2	5	420
Profit/unit Rs.	3	2	5	

Formulate the Linear programming model to determine the optimum daily production for three products, that maximize profit.

Sol:-

Let;

x_1 = No. of units of product 1.
x_2 = " " " " " 2.
x_3 = " " " " " 3

are produced.

The mathematical model of the problem is

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Max.

$$Z = 3x_1 + 2x_2 + 5x_3$$

S.t

$$3x_1 + 2x_3 \leq 430$$
$$x_1 + 4x_2 \leq 460$$
$$3x_1 + 2x_2 + 5x_3 \leq 420$$
$$x_1, x_2, x_3 \geq 0$$

2001
2006

Q:- The Apse Television Company has to decide on the no. of 27 and 20 inch sēls to be produced at one of its factories. Market research indicates that at 40 of the 27 inch sēls and 10 of the 20 inch sēls can be sold per month. The max. no. of work hrs. ~~atte~~ available is 500 per month. A 27-inch sēl requires 20 hrs. and 20 inch needs 10 work hours. Each 27 inch sēl sold producesses a profit of Rs- 12000 and each 20 inch sēl produces a profit of Rs- 8000. A wholesaler has agreed to purchased all the television sēls if the nos. does nōt exceeds the maxima indicāled by the markēl research.

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(127):-

a) Formulate a L.P model for this problem.
b) Use the graphical method to solve the problem.

Sol:-

a) Let

x_1 = No. of units of 27 inch set

x_2 = " " " " 20 " ".

are produced.

Then,

the mathematical model of the problem is given by:

Max.

$$Z = 12000x_1 + 8000x_2.$$

Subject to.

$$x_1 \leq 40.$$

$$x_2 \leq 10.$$

$$20x_1 + 10x_2 \leq 500.$$

$$x_1, x_2 \geq 0.$$

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b) Next, we solve it by Graphical Method.

Step I:

We convert the constraints into equality

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Q#8

Max.

$$Z = 12000x_1 + 8000x_2.$$

s.t.

$$x_1 = 40. \rightarrow (1)\ l_1$$

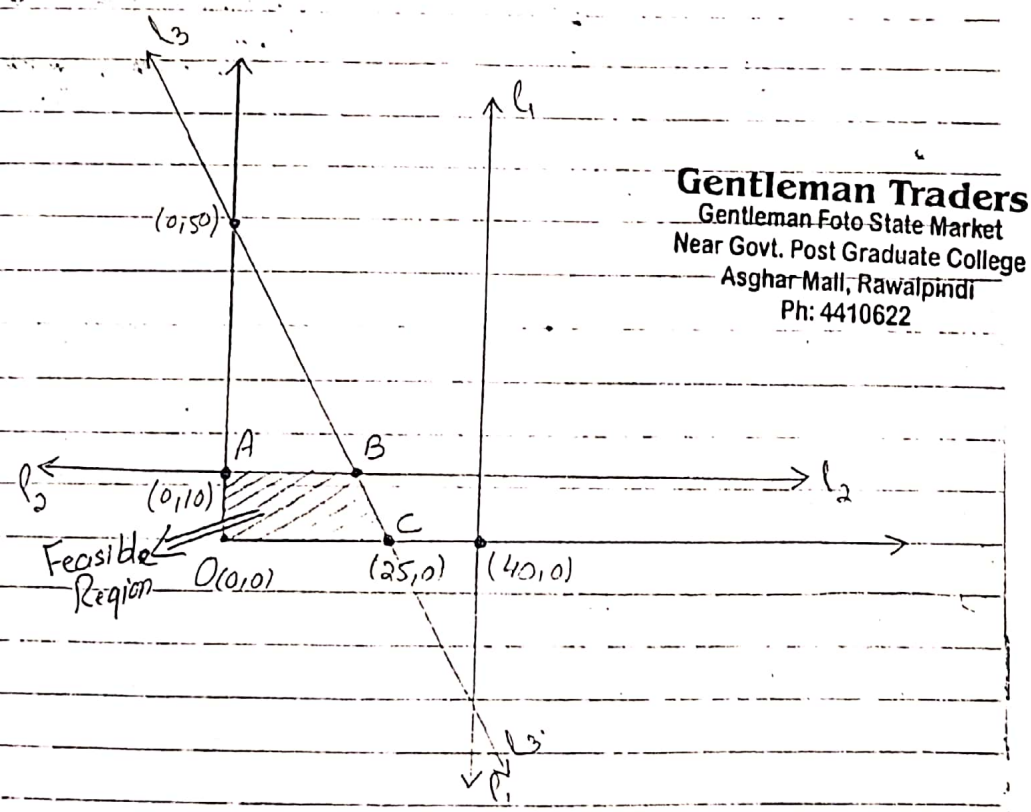
$$x_2 = 10 \rightarrow (2)\ l_2.$$

$$20x_1 + 10x_2 = 500$$

$$\Rightarrow \frac{1}{25}x_1 + \frac{1}{50}x_2 = 1 \rightarrow (3)\ l_3.$$

Step II

Draw the graph.



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(129).

Corner points of feasible region are O, A, B and C.

Here;

$O(0,0)$, $A(0,10)$, $B = ?$ $C(25,0)$.

For Point B;

We solve l_2 and l_3 we have

$$x_2 = 10$$

As;

$$20x_1 + 10x_2 = 500$$

$$20x_1 + 10(10) = 500$$

$$\Rightarrow x_1 = 20$$

So;

$B(20, 10)$.

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Step III;

As;

$$Z = 12000x_1 + 8000x_2$$

Using $O(0,0)$.

$$\Rightarrow Z = 0$$

Using $A(0,10)$.

$$\Rightarrow Z = 80,000$$

Using $B(20,10)$.

$$\Rightarrow Z = 3,20,000$$

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(132)

Using C(2,1,0).

$\Rightarrow Z = 3,00000$

Optimal Sol: is

$Z = 3,20,000$,
$x_1 = 20$.
$x_2 = 10$
} Ans:-

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Q8.11 A Tailor has the following material 16 square yard cotton, 11 sq. yard silk, and 15 square yard wool. As suit requires 2 sq. yard cotton, 1 sq. yard silk and 1 sq. yard wool.

A gown requires 1 sq. yard cotton, 2 sq. yard silk and 3 sq. yard wool.

If the suit sales for Rs. 300 and gown sales for Rs. 500.

How many garments should the tailor made to maximize the revenue.

Formulate the problem as an LP model and then solve by the graphical method.

Sol:-

Let;

x_1 = No. of units of suit.

x_2 = " " " " gown

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(131)

are produced.

The mathematical model of the problem is given as.

Max.

$$Z = 300x_1 + 500x_2$$

S/t

$$2x_1 + x_2 \leq 16$$
$$x_1 + 2x_2 \leq 11$$
$$x_1 + 3x_2 \leq 15$$
$$x_1, x_2 \geq 0.$$

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Next, we solve the problem by graphical method.

Step I

Convert the constraints into equality.

Max.

$$Z = 300x_1 + 500x_2.$$

S/t.

$$2x_1 + x_2 = 16$$
$$x_1 + 2x_2 = 11$$
$$x_1 + 3x_2 = 15$$

So,

$$\frac{1}{8}x_1 + \frac{1}{16}x_2 = 1 \longrightarrow (1)\ l_1$$
$$\frac{1}{11}x_1 + \frac{1}{11/2}x_2 = 1 \longrightarrow (2)\ l_2.$$
$$\frac{1}{15}x_1 + \frac{1}{5}x_2 = 1 \longrightarrow (3)\ l_3$$

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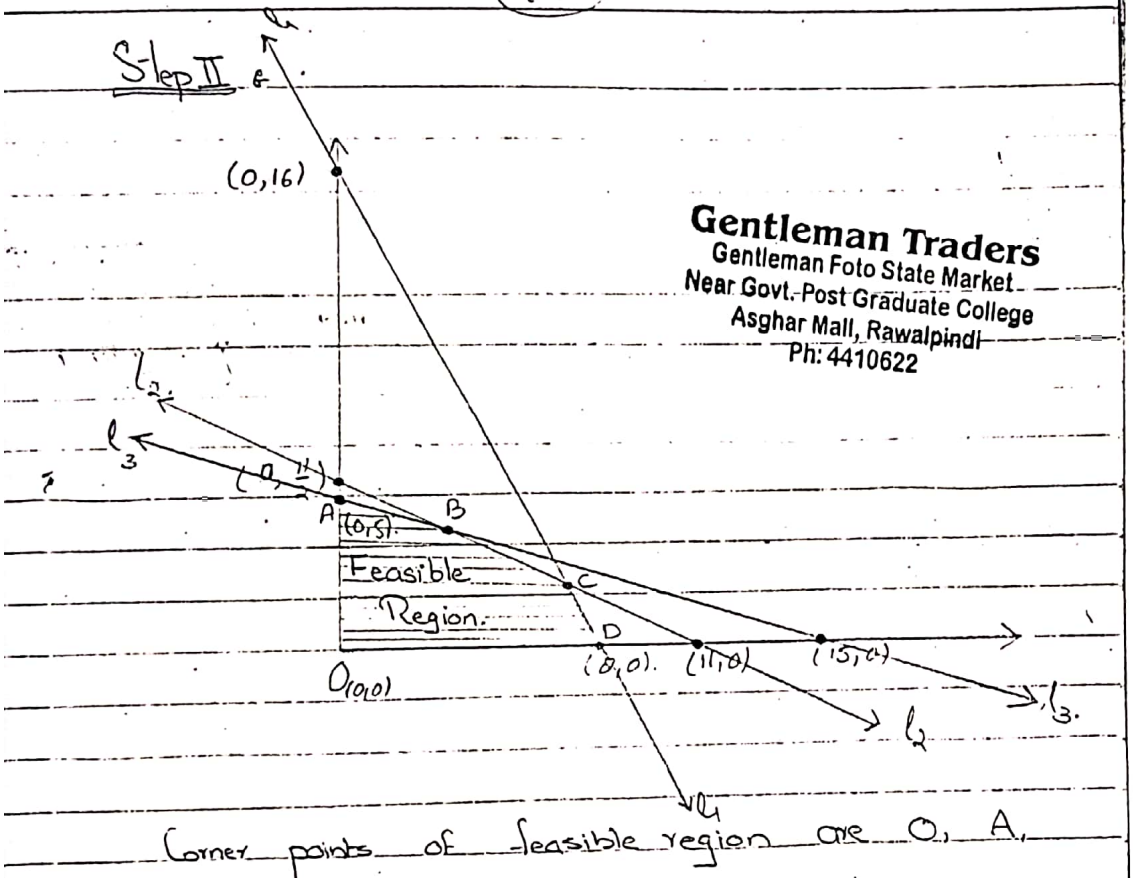
Step II



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Corner points of feasible region are O, A, B, C and D.

Here:

O(0,0), A(0,5), D(8,0), B = ?, C = ?

For Point B

For this we solve l_2 and l_3.

Subtracting ② and ③ we get

$$x_1 + 2x_2 = 11$$
$$\pm x_1 \pm 3x_2 = \pm 15$$
$$-x_2 = -4$$

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(133)

$\Rightarrow \quad x_2 = 4$

Putting in Eq ②

$x_1 + 2(4) = 11$

$x_1 + 8 = 11$

$\Rightarrow \quad x_1 = 3.$

So;

$B\ (3, 4).$

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For Point C &

For this we solve l_1 and l_2.

Xing Eq ① by 2 and then subtracting from Eq ②.

$4x_1 - 2x_2 = 32.$

$\pm x_1 \pm 2x_2 = \pm 11$

$3x_1 = 21$

$\Rightarrow \quad x_1 = 7.$

So; Eq ① gives.

$2(7) + x_2 = 16$

$\Rightarrow \quad x_2 = 2.$

$\Rightarrow \quad x_2 = 2.$

So;

$C\ (7, 2).$

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Step III :

As:

$Z = 300x_1 + 500x_2$

Using $O(0,0)$.

$\Rightarrow Z = 0$

Using $A(0,5)$.

$\Rightarrow Z = 2500$

Using $B(3,4)$.

$\Rightarrow Z = 2900$

Using $C(7,2)$.

$\Rightarrow Z = 3100$

Using $D(8,0)$.

$\Rightarrow Z = 2400$

Hence,

Optimal sol. is:

$Z = 3100$

$x_1 = 7$; $x_2 = 2$ } Ans:=

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SECTION II

:TRANSPORTATION PROBLEMS:-

Q:- Write a note on transportation problems?

Ans:- The transportation problem is the important class of Linnear Programming model. In this class the model seeks minimization of the cost of the transportation of a commodity from the number of source to several destination.

Example:-

Products may be transported from the factories to the retail stores. The objective is to determine the amount shipped from each source to the each destination that minimize the shipped cost while satisfying both the supply limit and the demands requirements.

The model assume that the shipping cost on the given route is directly proportional to the number of units shipped on that route. The general problem is represented as.

Suppose that there are m-sources and n-destinations. Let a_i be the no. of units supply are available at

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source i. such that $(i=1,2,3,----m)$ and let b_j be the number of units of demand at destination j. $(j=1,2,3,----n)$. Let C_{ij} be the per unit transportation cost enroute (i,j). Joining source i and destination j, the objective is to determine the no. of units transported from i to the destination j such that the net transportation cost is minimum.

Mathematical Model Of Transportation Problem:-

Let x_{ij} be the number of units shift from source i to the destination j, then the LP mode of this problem is ~~minimum minimize~~

Minimize.

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

objective function

Subject to.

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i=1,2,3----m.$$

(availbility Constraint)

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \cancel{\sum_{i=1}^{m} C_{ij} = b_j}$$

(Requirement Constraint) $j=1,2,3,-----n.$

This model can also be represented in tabular form as follows.

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Destinations

Source:	1	2	3	----	j	----	n	Supply.
1	C_{11}	C_{12}	C_{13}	----	C_{1j}	----	C_{1n}	a_1
2	C_{21}	C_{22}	C_{23}	----	C_{2j}	----	C_{2n}	a_2
3	C_{31}	C_{32}	C_{33}	----	C_{3j}	----	C_{3n}	a_3
i	C_{i1}	C_{i2}	C_{i3}	----	C_{ij}	----	C_{in}	a_i
m	C_{m1}	C_{m2}	C_{m3}	----	C_{mj}	----	C_{mn}	a_m
Demand	b_1	b_2	b_3	----	b_j	----	b_n	

Balance Transportation Model:-

If

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

i.e. Total Supply = Total Demand.

Then it is called as balance transportation model.

Unbalance Transportation Model:-

If

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$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

Total Supply ≠ Total Demand.

Then it is called an unbalance model.

Note:-

To solve an unbalance transportation model we first balance the model by adding a dummy source or dummy customer according to as supply is less than demand or greater than demand.

Transportation Algorithm Technique:-

The transportation algorithm technique follows the steps of the simplex method. However instead of using the regular simplex table, we take advantage of special structure of the transportation model to present. The algorithm in a more convenient form is as follows.

✓ **Step I:-**

Determine a starting feasible solution.

✓ **Step II:-**

Use the optimality condition of the simplex method to determine the entering variable.

✓ **Step III:-**

Use the feasibility condition of the

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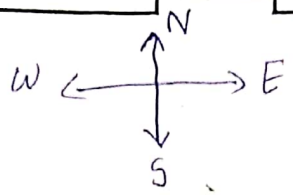

simpler method to determine the leaving variable from all the current basic variables. Repeat the process until all non-basic variables become zero or -ive.

Determination Of The Starting Solution:-

There are the five methods for finding the starting solution of the transportation model.

i) North West Corner Method
ii) Least Cost Method
iii) Row Minimum Method.
iv) Column Minimum Method.
v) Vogels Approximation Method.

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NOTE:-

Starting solution is not unique but optimal solution (if it exists) is always unique. Usually vogels approximation method gives the best starting solution.

Q:- Write a note on North-West Corner method. Also satisfy the name of its method.

Sol:- The method starts at the north west cell of the table that is why called north-west corner method.

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The method is as follows:

Step I:-

Allocate as much as possible to the selected cell and adjust the associated amount of supply and demand by subtracting the allocate amount.

Step II:-

Cross out the row and coulumn with zero supply or demand to indicate that no further assignment can be made in a row or coulumn if the both row or coulumn are zero. Simultaneously cross out only one and leave a zero supply or (demand) in the uncrossed row or (coulumn).

Step III:-

If exactly one row or coulumn is left uncrossed ~~been~~ then, stopped, otherwise move to the cell of the right, if a coulumn has just been crossed or the one below if a row has been crossed out go to the Step I.

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Q:-1 Find the starting solution of the following transportation ~~model~~ problem by N.W.C method.

Source \ Destination	1	2	3	4	Supply
1	10	0	20	11	15
2	12	7	9	20	25
3	0	14	16	18	5
Demand	5	15	15	10	

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Sol:-

Total Supply = 45

Total Demand = 45

Since;

Total Supply = Total Demand.

∴ Model is balance.

So,

We use the N.W.C method to find the starting solution.

10 (5)	0 (10)	20	11	~~15~~ ~~10~~
12	7 (5)	9 (15)	20 (5)	25 ~~20~~ ~~5~~
0	14	16	18 (5)	5
5	~~15~~ 5	~~15~~	~~10~~ =	

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Transportation cost is:

$5 \times 10 = 50$
$10 \times 0 = 0$
$5 \times 7 = 35$
$15 \times 9 = 135$
$5 \times 20 = 100$
$5 \times 18 = 90$
410 Ans:-

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Q2:- Find the starting solution of the following transportation problem by North-West corner method:

Source \ Destination	3	5	10	Supply
1				5
2	12	4	18	5
3	1	2	7	10
Demand	12	4	4	

Sol:-

Total Supply = 20
Total Demand = 20

Since,

Total Supply = Total Demand.

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∴ Model is balanced.

So,

We use N.W.C method to find the starting solution.

3 / 5	5	0	5
2 / 5	4	8	~~5~~
1 / 2	2 / 4	7 / 4	~~10~~ ~~8~~ ~~4~~
~~12~~ 7 ~~2~~	4	4	

Transportation Cost is.

$5 \times 3 = 15$

$5 \times 2 = 10$

$2 \times 1 = 2$

$4 \times 2 = 8$

$4 \times 7 = 28$

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(146)

Q5.11 Find the starting sol. of the following transportation problem by N.W.C method.

Source \ Destination				
	0	2	1	5 Supply.
	2	1	5	10
	2	4	3	5
Demand	5	5	10	

Sol:-

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Total Supply = 20
Total Demand = 20

Since:

Total Supply = Total Demand

∴ Model is balanced

Now, we apply N.W.C method to solve the problem.

	0	2	1	5 0
	5	0		
	2	1	5	10 5
		5	5	
	2	4	3	5
			5	
	5	5	10	5

So;

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(145)

Transportation Cost is.

$5 \times 0 = 0$

$0 \times 2 = 0$

$5 \times 1 = 5$

$5 \times 5 = 25$

$5 \times 3 = 15$

45 Ans:-

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Q:- LEAST COST METHOD:- (Cost Minimum Method)

The least cost method (L.C.M) finds a better starting solution by concentrating on the cheapest routes.

Instead of starting with NWC ~~method~~ cell we start by assigning as much as possible to the cell with the smallest unit cost. We then crossed-out the satisfies row or coulumn and adjust the amount of supply and demand accordingly. If both a row and a coulumn are satisfied simultaneously only one is crossed-out the same as in the N.W.C method.

Next, we always look for the uncrossed cell with the smallest unit cost and repeat the process until we are left at the end with

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exactly one uncrossed-out row or column.

Q1. Find the starting solution of the following transportation problem by L.C.Method.

Source	Destination			Supply
	0	2	1	5
	2	1	5	10
	2	4	3	5
Demand	5	5	10	

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Sol:-

Total Supply = 20
Total Demand = 20

Since:

Total Supply = Total Demand.

∴ Model is balanced.

Now we apply L.C.M. for the starting solution.

	10	12	11	5
5			0	
	12	11	15	5
		5	5	
	12	14	13	5
			5	
	5	5	10	
			5	

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(147)

So,

Transportation Cost is.

$5 \times 0 = 0$

$0 \times 1 = 0$

$5 \times 1 = 5$

$5 \times 5 = 25$

$5 \times 3 = 15$

45

Ans:-

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Q 2.11 Find the starting sol. of the following transportation problem by L.C method.

Source \ Destination					Supply
	10	6	20	11	15
	12	8	9	20	25
	6	14	16	18	5
Demand	5	15	15	10	

Sol:-

Total Supply = 45

Total Demand = 45

Since:

Total Supply = Total Demand.

∴ Model is balanced.

Now, we apply the L.C.M to solve find the starting solution.

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10	2 (15)	20	11	15
12	7 (0)	9 (15)	20 (10)	25 10
0	14	16	0 (15)	5 ∅
5				
5	15	15	10	

Transportation Cost is:

$0 \times 10 = 0$
$15 \times 0 = 0$
$5 \times 12 = 60$
$0 \times 7 = 0$
$15 \times 9 = 135$
$5 \times 20 = 100$
$10 \times 20 = 200$
$5 \times 18 = 90$
$0 \times 18 = 0$
385
335 Ans:-

Vogels Approximation Method (VAM)

VAM is the improved version of L.C. Method that generally provides the better starting solution

The steps of this method are as follows:

Step I:-

Evaluate a penalty for each row and each column by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost in the row (column).

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Step II :-

Identify row or column with largest penalty. Allocate as much as possible to the variable with the least unit cost in the selected row (column). If a row (column) are satisfied simultaneously only one of the two is crossed-out and the remaining row (column) is assigned zero.

Any row (column) with zero supply and demand should not be used in computing further penalties.

Step III :- (a)

If exactly one row (column) with zero supply or demand remains uncrossed-out, stop the process.

(b) :- If one row (column) with +ive supply (demand) remains uncrossed-out, determine the basic variable in the row (column) by L.C method.

(c) :- If all the uncrossed-~~out~~ rows (columns) have the zero supply and demand, determine the zero basic variable by L.C method.

(d) :- Otherwise go to the Step I.

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(58)

Q 1: Find the starting solution for the following transportation problem by VAM.

Source	Destination				Supply
	10	0	20	11	15
	12	7	9	20	25
	0	14	16	18	5
Demand	5	15	15	10	

Sol:

Total Supply = 45

Total Demand = 45

Since

Total Supply = Total Demand

∴ Model is balanced.

Now, we apply Vogels approximation method to find the starting solution.

				Row Penalty
10	0 (15)	20	11 (0)	15 ← 10 (11) 9
12	7	9 (15)	20 (10)	25 ← 2 2 (11)
0 (5)	14	16	18 (0)	5 ← (14) 2 2
5	15	15	10	

Column Penalty

10 7 7 7
7 7 7
0 7 7

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Transportation Cost is.

$15 \times 0 = 0$

$0 \times 11 = 0$

$15 \times 9 = 135$

$10 \times 20 = 200$

$0 \times 5 = 0$

$0 \times 10 = 0$

335 Ans:-

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Method Of Multipliers :- OR

U-V Method :- OR

Modi Method :-

Q1. Solve the following transportation problem starting with NWC-method

Source \ Destination				Supply
	0	2	1	5
	2	1	5	10
	2	4	3	5
Demand.	5	5	10	

Sol:-

Total Supply = 20

Total Demand = 20

Since,

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(Multipliers کی قدر Fill-box سے ہمیشہ اتنا زیادہ ہونی چاہیئے)

Total Supply = Total Demand.

∴ Model is balanced.

Now, we apply NWC method.

10	12	11	5 / 0
5	0		
12	11	15	10 / 5
	5	5	
12	14	13	5
		5	
5 / 5	5 / 5	10 / 5	

So;

$5 \times 0 = 0$

$0 \times 2 = 0$

$5 \times 1 = 5$

$5 \times 5 = 25$

$5 \times 3 = 15$

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Now, we apply multipliers method to improve this solution, so assign a multiplier to each row and column.

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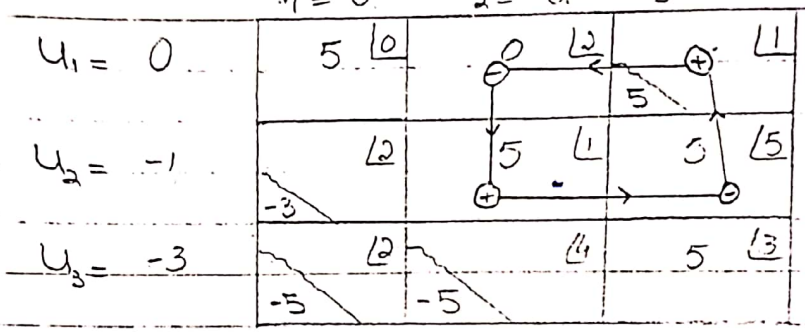
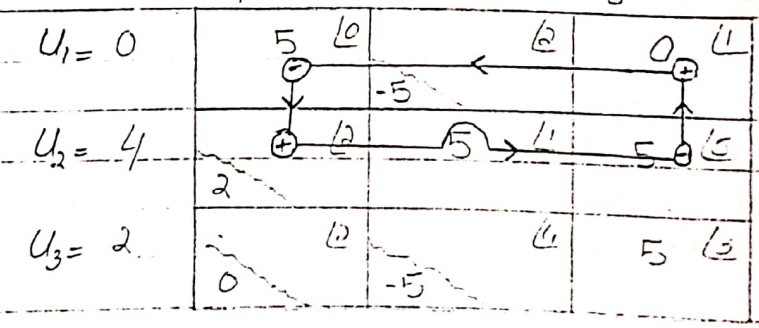
$V_1 = 0$ $V_2 = 2$ $V_3 = 6$



The minimum value in the -ive box is zero. Subtract this value from the -ive boxes, add in +ive boxes and leave the remaining unchanged.

Then again assign the multiplier to each row and coulumn.

$V_1 = 0$ $V_2 = -3$ $V_3 = 1$



The minimum value in the -ive bores is 5. Subtract this value from the -ive bores and add to the +ve boxes and leave the remaining unchanged.

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Then, again assign multipliers to each row and coulumn.

	$V_1 = -2$	$V_2 = -3$	$V_3 = 1$
$U_1 = 0$	10 (-2)	12 (-5)	5 11
$U_2 = 4$	5 12	5 11	0 15
$U_3 = 2$	12 (-2)	14 (-5)	5 13

Since, all the shadow cost are either zero or -ve. So, this is the optimal table and optimal sol. is.

$5 \times 1 = 5$
$5 \times 2 = 10$
$5 \times 1 = 5$
$0 \times 5 = 0$
$5 \times 3 = 15$
35 Ans:-

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Q-2:- Find the most economical transportation cost in the following model starting with NWC method.

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$0	$2	$1	6
$2	$1	$5	9
$2	$4	$3	5
5	5	10	

Sol:-

Total Supply = 20

Total Cost = 20

Since,

Total Supply = Total Cost

∴ Model is balanced

Now, we apply NWC method to find the starting solution.

10	12	11	6
5	1		
12	11	15	9 5
	4	5	
12	12	5 13	5
5	5 4	15 5	

Transportation Cost is.

5 × 0 = 0

1 × 2 = 2

4 × 1 = 4

5 × 5 = 25

5 × 3 = 15

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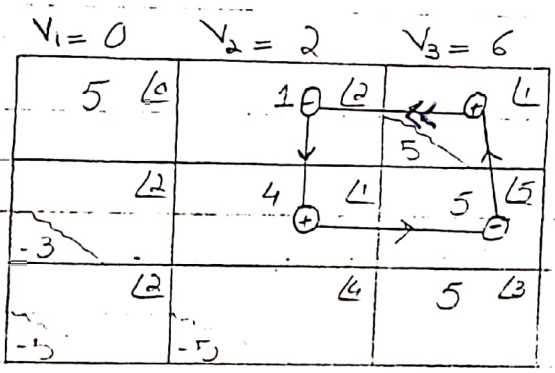
Now, we apply multipliers method to improve this solution.

So, assign a multiplier to each row and each column.



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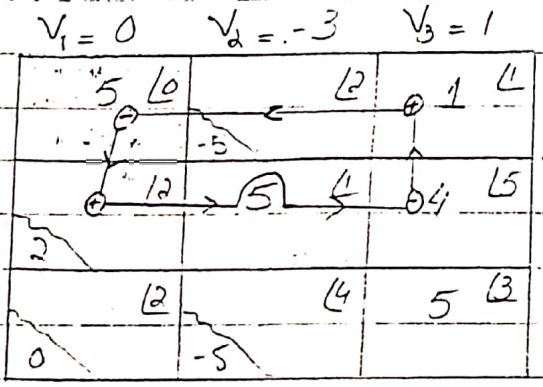
The minimum value in the -ive boxes is 1. Subtract this value from the -ive boxes, add this value to the +ive boxes and leave the remaining unchanged.

Then, again assign a multiplier to each row and each column.



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The minimum value in the -ive boxes is ~~zero~~ 4. Subtract this value to -ive boxes and add this value to +ive boxes and leave the remaining unchanged.

Then, assign the multipliers to each row and each coulumn again.

	$V_1 = 0$	$V_2 = -1$	$V_3 = 1$
$U_1 = 0$	1 (0)	(2)	5 (1)
		-3	
$U_2 = 2$	1 (2)	5 (1)	(3)
			-2
$U_3 = 2$	(2)	(4)	5 (3)
	0	-3	

Since all the shadow cost are either zero or -ive. So, this is the optimal table and optimal sol. is.

$1 \times 0 = 0$
$5 \times 1 = 5$
$4 \times 2 = 8$
$5 \times 1 = 5$
$5 \times 3 = 15$
33 Ans:-

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Q:-3. Solve the following transportation problem ~~by~~ ~~L.~~ starting with L.C.M.

Source	Destination			Supply
	$0	$2	$1	6
	$2	$1	$5	9
	$2	$4	$3	5
Demand	5	5	10	

Sol:-

Total Supply = 20

Total Demand = 20

Since

Total Supply = Total Demand.

∴ Model is balanced.

Now, we apply L.C.M. to find the starting solution.

0 / 5	2	1 / 1	~~6~~ ~~1~~
2	1 / 5	5 / 4	~~9~~ ~~4~~
2	4	3 / 5	~~5~~
~~5~~	~~7~~	~~10~~ ~~9~~ ~~5~~	

So;

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Transportation Cost is

$5 \times 0 = 0$

$5 \times 1 = 5$

$1 \times 1 = 1$

$4 \times 5 = 20$

$5 \times 5 = 15$

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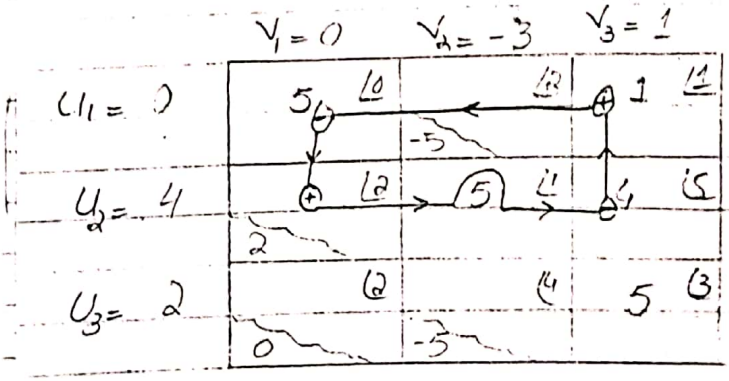
Now, we apply the multiplier method to improve the solution. So assign a multiplier to each row and coulumn.



The minimum value in the -ve boxes is 4. Subtract this value from -ve boxes and add to the +ive boxes and leave the remaining unchanged.

Then, assign a multiplier to each row and coulum.

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	$V_1 = 0$	$V_2 = -1$	$V_3 = 1$
$U_1 = 0$	1 ⌊0	⌊2 (−3)	5 ⌊1
$U_2 = 2$	4 ⌊2	5 ⌊1	⌊5 (−2)
$U_3 = 2$	⌊2 (0)	⌊4 (−3)	5 ⌊3

Since, all the shadow cost are either zero or -ive, so this is the optimal table and the optimal solution is.

$1 \times 0 = 0$
$5 \times 1 = 5$
$4 \times 2 = 8$
$5 \times 1 = 5$
$5 \times 3 = 15$
33 Ans:-

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Assignment

Q4. Solve the following transportation problem starting with LCM.

Source	Destination			Supply
	0	2	1	5
	2	1	5	10
	2	4	3	5
Demand	5	5	10	

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(161)

Sol:-

Total Supply = 20
Total Demand = 20.

Since,

Total Supply = Total Demand.

∴ Model is balanced.

Now, we apply the LC Method to find the starting solution.

	5 (0)	(2)	0 (1)	5
	(2)	5 (1)	5 (5)	10
	(2)	(4)	5 (3)	5
	5	5	10	5

Transportation cost is:

$5 \times 0 = 0$
$5 \times 1 = 5$
$0 \times 1 = 0$
$5 \times 5 = 25$
$5 \times 3 = 15$
45

Now we apply the multiplier method to improve this solution. So assign 0 multiplier to each row

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and each column.

	$V_1 = 0$	$V_2 = -3$	$V_3 = 1$
$U_1 = 0$	5 (10)	(12)	0 (1)
$U_2 = 4$	(12)	5 (1)	5 (5)
$U_3 = 2$	(12)	(4)	5 (3)

The minimum value in the -ive boxes is 5. Subtract this value from -ive boxes, add to the +ive boxes and leave the remaining boxes unchanged.
Then again assign a multiplier to each row and each column.

	$V_1 = -2$	$V_2 = -3$	$V_3 = 1$
$U_1 = 0$	(10)	(12)	5 (1)
$U_2 = 4$	5 (12)	5 (1)	0 (5)
$U_3 = 2$	(12)	(4)	5 (3)

Since all the shadow cost are either zero or -ive. So, this is the optimal

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(163)

-lable and optimal sol. is:

$5 \times 1 = 5$
$5 \times 2 = 10$
$5 \times 1 = 5$
$0 \times 5 = 0$
$5 \times 3 = 15$
35

Ans:-

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Row- Minimum Method:-

In LCM we made the first allocation in the cell with minimum transportation cost over the whole matrix. But, here only the first row minimum cost is reached for the initial solution. This allocation is repeated for the other cell in the first row till supply at the first source is exhausted. Then by one by one all the rows exhausted.

Q:-1/ Find the starting sol. of the following transportation problem by the RMM.

5	8	3	6	30
4	5	7	4	50
6	2	4	6	20
30	40	20	10	

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(164)

Sol:-

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Total Supply = 100
Total Demand = 100
Since, Total Supply = Total Demand.

∴ Model is balanced.
Now, apply the row minimum method to find the starting sol. of the following transportation problem.

	10 (5)	(8)	20 (3)	(6)	~~30~~ 10
	20 (4)	20 (5)	(7)	10 (4)	~~50~~ ~~30~~ ~~20~~
	(6)	20 (2)	(4)	(6)	~~20~~
	~~20~~	~~30~~ 30	40	~~20~~	~~10~~

Transportation Cost is:

$10 \times 5 = 50$
$20 \times 3 = 60$
$20 \times 4 = 80$
$20 \times 5 = 100$
$20 \times 2 = 40$
$10 \times 4 = 40$
$= 370$ Ans:-

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Q2-11 Find the most economical sol. of the following transportation problem starting with row minimum method.

0	2	1	6
2	1	5	9
2	4	3	5
5	5	10	

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Total Supply = 20
Total Demand = 20

Since,
Total Supply = Total Demand,
∴ Model is balanced.

First, we find the starting solution by row minimum method.

5 (0)		(2)	1 (1)	6 X
(2)	5 (1)		4 (5)	9 4
(2)		(4)	= (3)	5
5	5	10	9	3

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Transportation Cost is.

$5 \times 0 = 0$
$1 \times 1 = 1$
$5 \times 1 = 5$
$4 \times 5 = 20$
$5 \times 3 = 15$
41

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Now, we apply the multiplier method to improve this sol. So assign a multiplier to each row and each coulumn.

	$V_1 = 0$	$V_2 = -3$	$V_3 = 1$
$u_1 = 0$	5 [0] (−)	[2] −5	1 [1] (+)
$u_2 = 4$	(+) [2] 2	5 [1]	4 [5] (−)
$u_3 = 2$	[2] 0	[4] −5	5 [3]

Since the minimum value in the -ive boxes is 4. So, subtract this value from -ive boxes and add to the +ive boxes and leave the remaining unchanged. Then again assign the multipliers to each row and each coulumn.

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	$V_1 = 0$	$V_2 = -1$	$V_3 = 1$
$U_1 = 0$	1 [10]	[12]	5 [11]
$U_2 = 2$	4 [12]	5 [11]	[15]
$U_3 = 2$	[10]	[4]	5 [13]

-3, -2, 0, -3

Since, the shadow cost are either zero or -ive. So, this is the optimal table and optimal sol. is

$1 \times 0 = 0$
$5 \times 1 = 5$
$4 \times 2 = 8$
$5 \times 1 = 5$
$5 \times 3 = 15$
33

Ans:-

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Column Minimum Method:-

Q:- 1.) Find the starting solution of the following transportation problem by column minimum method.

5	8	3	6	30
4	5	7	4	50
6	2	4	6	20
30	40	20	10	

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(168)

Sol:-

Total Supply = 20
Total Demand = 20

Since,

Total Supply = Total Demand.

∴ Model is balanced.

Now, we apply the Coulumn minimum method to find the starting Sol.

	5	18	20 13	10 16
	30 14	20 15	17	0 14
	16	20 12	14	16

Transportation Cost is

30 x 4 = 120
20 x 2 = 40
20 x 5 = 100
20 x 3 = 60
0 x 4 = 0
10 x 6 = 60
380 Ans:-

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Q2.11 Find the most economical cost in the following problem starting with Column minimum method.

0	2	1	6
2	1	5	4
2	4	3	5
5	5	10	

Sol:-

Total Supply = 20
Total Cost = 20

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Since,

Total Supply = Total Cost

∴ Model is balanced.

Now we find the starting sol. by using Column minimum method.

5 (10)	(12)	1 (11)	6 1
(12)	5 (1)	4 (15)	9 4
(12)	(14)	5 (13)	5
5	5	10	
			9 4

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(170)

Transportation Cost is.

$5 \times 0 = 0$

$5 \times 1 = 5$

$1 \times 1 = 1$

$4 \times 5 = 20$

$5 \times 3 = 15$

41

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Now, we apply the multiplier method to improve this solution. So assign a multiplier to each row and coulumn.

	$V_1 = 0$	$V_2 = -3$	$V_3 = 1$
$U_1 = 0$	5 (10)	(12)	1 (11)
$U_2 = 4$	(12) 2	5 (11)	4 (15)
$U_3 = 2$	(12) 0	(4) -5	5 (3)

The minimum value in -ive boxes is 4.
Subtract this value to -ive box and add to the +ive boxes and left the remaining unchanged.

Then, again assign a multiplier to each row and each coulumn.

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	$V_1 = 0$	$V_2 = -1$	$V_3 = 1$
$U_1 = 0$	1 (10)	(2)	5 (11)
		-3	
$U_2 = 2$	4 (12)	5 (11)	(15)
			-2
$U_3 = 2$	(12)	(14)	5 (13)
	0	-3	

Since the shadow cost are either -ive or zero. So, this is the optimal table and the optimal sol is:

$1 \times 0 = 0$
$5 \times 1 = 5$
$4 \times 2 = 8$
$5 \times 1 = 5$
$5 \times 3 = 15$
33 Ans:-

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Unbalanced Transportation Problem:-

Imp

Q.11 In the following problem, the total demand exceeds the total supply. Suppose that the penalty cost per unit of unsatisfied demand are 5 ₹, 3 ₹ and 2 ₹ for the destination 1, 2 and 3 respectively. Determine the optimal sol. starting with VAM.

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5	1	7	10
6	4	6	80
3	2	5	15
75	20	50	

Sol:-

Total Supply = 105

Total Demand = 145

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Since

Total Supply ≠ Total Demand

∴ The model is not balanced.

First, we balance the model by introducing a dummy source whose supply is 40 units.

After to this we solve the problem by VAM

				Row Penalty		
5	10 1	7	~~10~~ ← (4)			
60 6	10 4	10 6	~~80~~ ~~70~~ ~~10~~	2	2	2
15 3	2	5	~~15~~	1	1	1
5	3	40 2	~~40~~	1	1	
~~60~~ ~~75~~	~~20~~ ~~10~~	~~50~~ ~~10~~				
Coulumn 2	1	3				
Penalty 2	1	(3)				
(3)	2	1				

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Transportation Cost is

$60 \times 6 = 360$

$15 \times 3 = 45$

$10 \times 1 = 10$

$10 \times 4 = 40$

$10 \times 6 = 60$

$40 \times 2 = 80$

595

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Now, we apply the multiplier method to improve this solution. So assign a multiplier to each row and each column.

	$V_1 = 3$	$V_2 = 1$	$V_3 = 3$	
$U_1 = 0$	15 (-2)	10 11	17 (-4)	~~15~~
$U_2 = 3$	60 16	10 14	10 16	~~26~~
$U_3 = 0$	15 13	12 (-1)	15 (-2)	~~15~~
$U_4 = -1$	15 (-3)	13 (-3)	40 12	~~40~~
	~~35~~	~~20~~	~~55~~	

Since all the shadow costs are either +ive or zero. So, this is the optimal table and the optimal sol. is 595.

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(174) mistake

Q2.11 Solve the following unbalanced transportation problem starting with VAM. The demand at destination 1 must be shift out from the source four.

5	1	0	20
3	2	4	10
7	5	2	15
9	6	0	15
5	10	15	

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Sol:-

Total Supply = 60

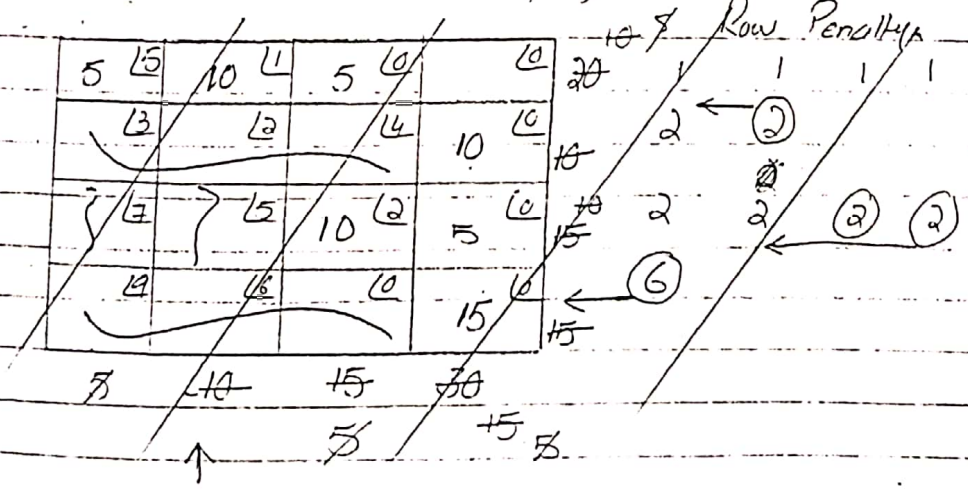
Total Demand = 30

Since

Total Supply ≠ Total Demand.

So, the model is unbalanced.

First we balance the model by introducing a dummy customer whose demand is 30 units. Then, we apply the VAM.



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Coulumn 2 1 2 ∅
Penalty 2 1 2 0
2 ↑(4) 2 0
2 2 0

Transportation Cost is:

5 x 5 = 25
10 x 1 = 10
5 x 0 = 0
0 x 0 = 0
5 x 0 = 0
10 x 2 = 20
15 x 0 = 0
= ~~55~~

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Raw Penalty.

	5	10 11	15	0	0	10 20
	3	2	4	10	0	10
	7	5	5	2	10	0 5 10
5	9	6	0	10	0	5 10

1 1 1 1
2 ← (2)
2 2 2 2
← (6)

5 10 5 5 30 20 10

Coulumn 2 1 2 0
Penalty 1 2 0
(4) 2 0
(2) 0

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Transportation Cost is:

$5 \times 9 = 45$

$10 \times 1 = 10$

$10 \times 0 = 0$

$5 \times 2 = 10$

$10 \times 0 = 0$

$10 \times 0 = 0$

$10 \times 0 = 0$

$= 65$

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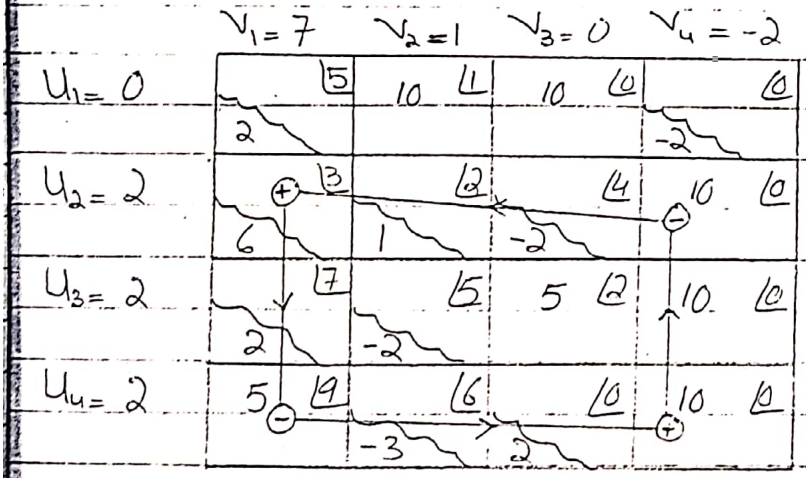
Now, to improve this solution we apply the multiplier method. So assign a multiplier to each row and each column.



The minimum value in the -ive boxes is 5. So, subtract this value from -ive box, add to +ive boxes and leave the remaining unchanged. Then, again assign multiplier to each row and each column.

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	$V_1=1$	$V_2=1$	$V_3=0$	$V_4=-2$
$U_1=0$	5 (-4)	10 11	10 10	10 (-2)
$U_2=2$	5 13	12 (1)	14 (-2)	5 10
$U_3=2$	17 (-4)	15 (-2)	5 12	10 10
$U_4=2$	19 (-6)	16 (-3)	10 (2)	15 10

The minimum value in -ive boxes is 5. Subtract this value to the -ive boxes, add to +ive boxes and leave the remaining unchanged. Then, again assign multipliers to each row and each column.

	$V_1=3$	$V_2=1$	$V_3=0$	$V_4=0$
$U_1=0$	5 (-2)	10 11	10 10	10 (0)
$U_2=0$	5 13	12 (-1)	14 (-4)	5 10
$U_3=0$	17 (-4)	15 (-4)	12 (-2)	15 10
$U_4=0$	19 (-6)	16 (-5)	5 10	10 10

Since all shadow costs are either zero or -ive. So, this is the

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optimal table and optimal sol: is.

$10 \times 1 = 10$
$10 \times 0 = 0$
$5 \times 3 = 15$
$5 \times 0 = 0$
$15 \times 0 = 0$
$5 \times 0 = 0$
$10 \times 0 = 0$

$= 25$ Ans:-

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Assignment Problems:-

Q:- Write a note on assignment problem.

Ans:- Another special of LP model is the assignment problem. It is actually a special case of more general transportation problem. Like the transportation ~~problem~~ method, the assignment is computationaly more efficient, than the simplex method. For a special type of LP model, it is used when there are n-jobs to be assigned to n-people. In such a way some measurements of effectiveness is optimized i-e some cost is minimize or some profit is maximize.

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Q:- Write a note on Hungarian assignment method.

Ans:-

Hungarian assignment method is used to solve assignment problem. This method consists of the following steps.

Step I:-

Subtract the smallest ~~minimum~~ element of the each row from every element of corresponding row.

Step II:-

Subtract the smallest element of each column from the corresponding column.

Step III:-

Draw the minimum number of the horizontal and vertical lines to cross all zeros.

Step IV

If the no. of lines ℓ drawn say (N) is equal to the order of the original matrix say (n) i-e N = n, then we get the one optimal assignment.

If N < n, select ~~a~~ set the lowest cost among all the elements of the reduced matrix. (Matrix obtained by performing Step I and Step II) which are not covered by the horizontal and vertical lines drawn. Subtract the

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smallest cost from all the uncovered elements of the reduced matrix and add it to the intersection of lines.

The other elements remain unchanged.

Repeat Step III to obtain a new reduced matrix.

Step V :-

Once, $n = N$, we find the optimal assignment as described below:

Examine the row one by one ~~until~~ until a row with exactly one single zero is found. Mark this zero by "□" to make as assignment. Then cross all the zeros, lying in the column maked by zero showing that assignment. Continue this till all the rows has been examined.

Step VI :-

Repeat Step V until one of the following situations arises:

i) No unmarked zero is left. This is the end of the algorithm.

ii) There lies ~~on~~ more than one unmarked zero in any row or column. Then mark "□" one of the unmarked zero arbitrarly and mark a cross (X) in the cells of the remaining zero in its row and column

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Repeat the process until no unmarked zero is left.

Thus exactly one marked zero will be obtained in each row in the optimal assignment.

Q:-1. Solve the following assignment.

(Worker)	(Machine) I	II	III
I	5	7	9
II	14	10	12
III	15	13	16

Sol:-

Step I:

Subtract the smallest element of each row from the corresponding row.

0	2	4	$P_1 = 5$
4	0	2	$P_2 = 10$
2	0	3	$P_3 = 13$

Step II:

Subtract the smallest element of each coulumn to the corresponding coulumn.

0	2	2
4	0	0
2	0	1

$q_{v_1} = 0$ $\quad q_{v_2} = 0$ $\quad q_{v_3} = 2$

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(18)

Step III :

Draw the min. horizental and vertical lines to cross all the zeros.

0	2	2
4	0	0
2	0	1

Step IV :

No. of lines drawn = Order of original matrix

So, it is the optimal table

Step V :

Complete the assignment

	I	II	III
I	[0]	2	2
II	4	~~0~~	[10]
III	2	[0]	1

Assignment is completed as:

I → I

II → III

III → II

and the cost is:

5 + 13 + 12 = 30 Ans:-

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(182)

Q2. Solve the following assignment.

	I	II	III	IV
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

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Sol:-

Step I:

Subtract the smallest element of each row from the corresponding row.

1−1 = 0	4−1 = 3	6−1 = 5	3−1 = 2	$P_1 = 1$
9−7 = 2	7−7 = 0	10−7 = 3	9−7 = 2	$P_2 = 7$
4−4 = 0	5−4 = 1	11−4 = 7	7−4 = 3	$P_3 = 4$
8−5 = 3	7−5 = 2	8−5 = 3	5−5 = 0	$P_4 = 5$

Step II:

Subtract the smallest element of each coulumn from the corresponding coulumn.

0	3	2	2
2	0	0	2
0	1	4	3
3	2	0	0

$q_1 = 0$ $q_2 = 0$ $q_3 = 3$ $q_4 = 0$

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Step III:

Draw the min no. of the horizontal and vertical lines to cross the zeros.

0	3	2	2
2	0	0	2
0	1	4	3
3	2	0	0

Step IV:

No. of lines drawn < Order of original matrix.

So, this is not an optimal table.

Now the smallest uncovered element is 1.

Subtract this element from the uncovered elements; add this element to the elements at the intersection of the lines and leave the remaining unchanged.

Then, again draw the min. no. of the horizontal and vertical lines to cross all the zeros.

0	2	1	1
3	0	0	2
0	0	3	2
4	2	0	0

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Now,

No. of the lines drawn = Order of the matrix (4)

So, this is an optimal table.

Step V:

Complete the assignment.

	I	II	III	IV
I	[0]	2	1	1
II	3	~~0~~	[0]	2
III	~~0~~	[0]	3	2
IV	4	2	~~0~~	[0]

Assignment is completed as

I → I

II → III

III → II

IV → IV

and cost is

1 + 10 + 5 + 5 = 21. Ans:-

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Q 3.11 Solve the following assignment problem.

	I	II	III	IV
I	5	5	-	2
II	~~7~~	4	2	3
III	9	3	5	-
IV	7	2	6	7

Sol:-

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(186)

Step I :

Subtract the smallest element from each row to the corresponding row.

3	3	–	0	$P_1 = 2$
5	2	0	1	$P_2 = 2$
6	0	2	–	$P_3 = 3$
5	0	4	5	$P_4 = 2$

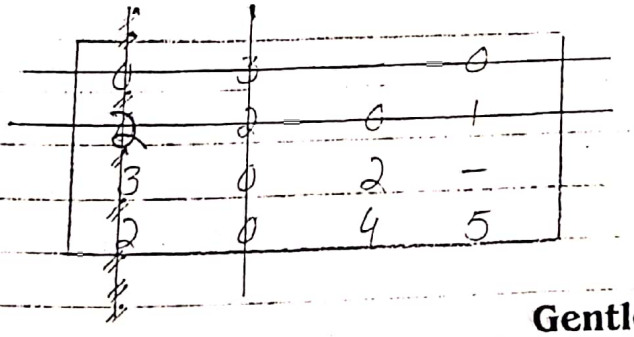
Step II :

Subtract the smallest element from each column to the corresponding column.

0	3	–	0
2	2	0	1
3	0	2	–
2	0	4	5

Step III : $a_1 = 3$ $a_2 = 0$ $a_3 = 0$ $a_4 = 0$

Draw the min no: of the horizental and vertical lines to cross all the zeros.



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(187)

Since;

No. of drawn lines < Order of original Matrix

So, this is not an optimal table.

The smallest uncovered element is "2".

Subtract this element from the uncovered element, add to the elements in the intersection of the lines and leave the remaining elements unchanged.

Then again drawn the horizental and vertical lines to cross all the zeros.

	I	II	III	IV
I	~~0~~	5	–	[0]
II	2	4	[0]	1
III	1	[0]	~~0~~	–
IV	[0]	~~0~~	2	3

Assignment is completed as.

I → IV

II → III

III → II

IV → I

Cost is as.

2 + 2 + 3 + 7 = 14 Ans:-

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Q4.4 (Assignment)

Solve the following assignment model.

	I	II	III	IV	V
I	3	8	2	10	3
II	8	7	2	9	7
III	6	4	2	7	5
IV	8	4	2	3	5
V	9	10	6	9	10

Sol:-

Step I:-

Subtract the smallest element of each row from the corresponding row.

1	6	0	8	1	$P_1 = 2$
6	5	0	7	5	$P_2 = 2$
4	2	0	5	3	$P_3 = 2$
6	2	0	1	3	$P_4 = 2$
3	4	0	3	4	$P_5 = 6$

Step II:- Subtract the smallest element of from each column to the corresponding column.

0	4	0	7	0
5	3	0	6	4
3	0	0	4	2
5	0	0	0	2
2	2	0	2	3

$q_1 = 1$ $q_2 = 2$ $q_3 = 0$ $q_4 = 1$ $q_5 = 1$

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Step III :

Draw the minimum number of the horizental and vertical lines to cross all the zeros.

0	4	0	7	0
5	3	0	6	4
3	0	0	4	2
5	0	0	0	2
2	2	0	2	3

Since,

No. of lines drawn < Order of the original Matrix

So, this is not an optimal table.

The smallest uncovered element is "2"

Subtract this element from the uncovered elements, add to the elements in the intersection of the lines, and leave the remaining element unchanged.

Then again draw the horizental and the verticcl lines to cross all the zeros.

0	4	2	7	0
3	1	0	4	2
3	0	2	4	2
5	0	2	0	2
0	0	0	0	1

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(80)

Now;

No. of lines drawn = Order of Origional Matrix.

Step IV:

Complete the assignment

	I	II	III	IV	V
I	~~0~~	4	2	7	[0]
II	3	1	[0]	4	2
III	3	[0]	2	4	2
IV	5	~~0~~	2	[0]	2
V	[0]	~~0~~	~~0~~	~~0~~	1

Assignment is completed as

I ⟶ V

II ⟶ III

III ⟶ II

IV ⟶ IV

V ⟶ I

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and the cost is

$3 + 2 + 4 + 3 + 9 = 21$ Ans:-

Q5. Solve the following assignment model.

	I	II	III	IV
I	50	50	—	20
II	70	40	20	30
III	90	30	50	—
IV	70	20	60	70
V	60	45	30	80

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Sol:-

Given model is unbalanced first we balance the model.

	I	II	III	IV	V
I	50	50	–	20	0
II	70	40	20	30	0
III	90	30	50	–	0
IV	70	20	60	70	0
V	60	45	30	80	0

Step I:

Subtract the smallest element of each row from the corresponding row. Since "0" is the smallest element is each row. So.

50	50	–	20	0
70	40	20	30	0
90	30	50	–	0
70	20	60	70	0
60	45	30	80	0

Step II:

Subtract the smallest element of each column from the corresponding column.

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0	30	–	0	0
20	20	0	10	0
40	10	30	–	0
20	0	40	50	0
10	25	10	60	0

$v_1=50$ $v_2=20$ $v_3=20$ $v_4=20$ $v_5=0$

Step III :-

Draw the min. no. of the horizental and vertical line to cross all the zeros.

0	30		0	0
20	20	0	10	0
40	10	30	–	0
20	0	40	50	0
10	25	10	60	0

No. of the lines drawn < Order of the orginal matrix

Since the smallest element in the uncovered elements is "10". Subtract this element from the uncovered elements, add to the elements in the intersection of the lines and leave the remaining unchanged.

Then again draw the min. no. of the horizental and vertical lines to cross all the zeros.

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0	30		0	10
20	20	0	10	10
30	0	20	–	0
20	0	40	50	10
0	15	0	50	0

Now

No. of the lines drawn = Order of the original Matrix.

So, this is an optimal table.

Step V &

Complete the assignment.

	I	II	III	IV	V
I	~~0~~	30	–	[0]	10
II	20	20	[0]	10	10
III	30	~~0~~	20	–	[0]
IV	20	[0]	40	50	10
V	[0]	15	~~0~~	50	~~0~~

Assignment is completed as:

I → IV

II → III

III → V

IV → II

V → I

Cost is given as:

20 + 20 + 0 + 20 + 60 = 120 Ans.-

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(CS4)

Q:- Write a note on Network minimization.

Net-Work Minimization:-

NWM deals with the determination of the branches that can be linked all the nodes of a network. Such that the length of the choosen branches are minimize. This minimum network is called minimum spanning -tree.

Procedure For Finding Minimal Spanning Tree:-

The procedure for finding the minimal spanning -tree is as follows:

Begin with any node and join it to closed node in the network. The resulting two nodes form a "connected set" and the remaining nodes unconnected sets.

Next choose, a node from the unconnected sets. i-e Nearest to any node in the connected set and add to the connected set.

The process will repeated until the connected set includes all the nodes of the network.

Q1.// Find the minimal spanning -tree in the following network:

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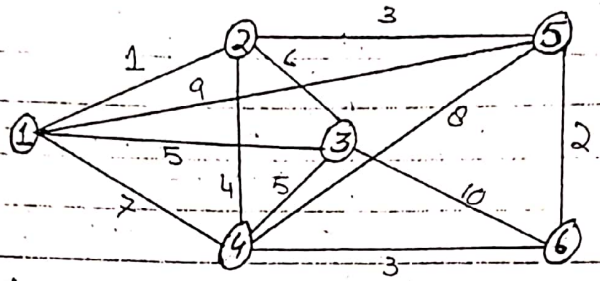
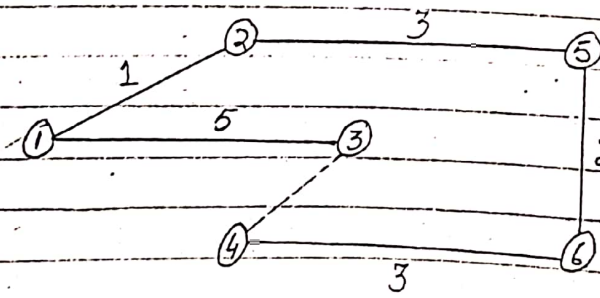


Sol:-

①		②		③	
1	1 — 2	2	1 — 1	3	5 — 1
1	5 — 3	2	6 — 3	3	6 — 2
1	7 — 4	2	4 — 4	3	5 — 4
1	9 — 5	2	3 — 5	3	10 — 6

④		⑤		⑥	
4	7 — 1	5	9 — 1	6	10 — 3
4	4 — 2	5	3 — 2	6	3 — 4
4	5 — 3	5	8 — 4	6	2 — 5
4	8 — 5	5	7 — 6		
4	3 — 6				

The minimal spanning tree is:



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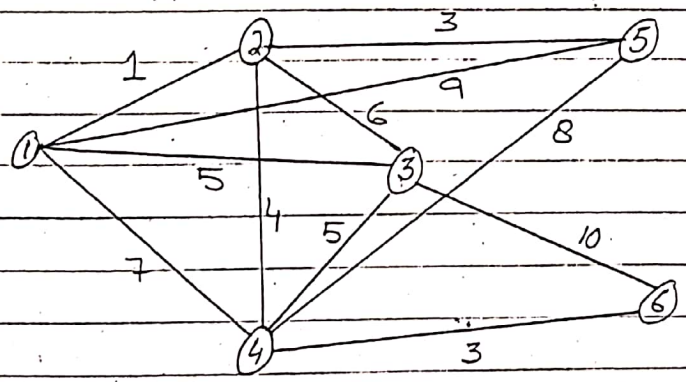
and cable used is.

$1 + 3 + 2 + 3 + 5 = 14$ Ans:-

Q2# The mid west T.V cable company is in the process of providing a cable servers to new five housing development areas. The cable system network ~~is~~ system sumarized in the fig below. The cable miles are shown on each branch. Node 1 represents the T.V station and the remaining node represents the new housing areas. It is required to determine the linkes, that will result in the use of minimum cable while all the areas are connected.



Sol:-

The minimal spanning is.

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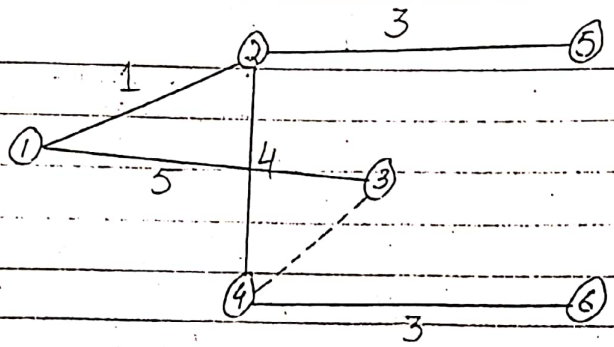
(L97)



①	②	③
~~1 1 2~~	~~2 1 1~~	~~3 5 1~~
~~1 5 3~~	2 6 3	3 6 2
1 7 4	~~2 4 4~~	3 5 4
1 9 5	~~2 3 5~~	3 10 6

④	⑤	⑥
4 7 1	5 9 1	6 10 3
~~4 4 2~~	~~5 3 2~~	~~6 3 4~~
4 5 3	5 8 4	
4 8 5		
~~4 3 6~~		

and cable used is.

$1 + 3 + 5 + 4 + 3 = 16$ Ans:-

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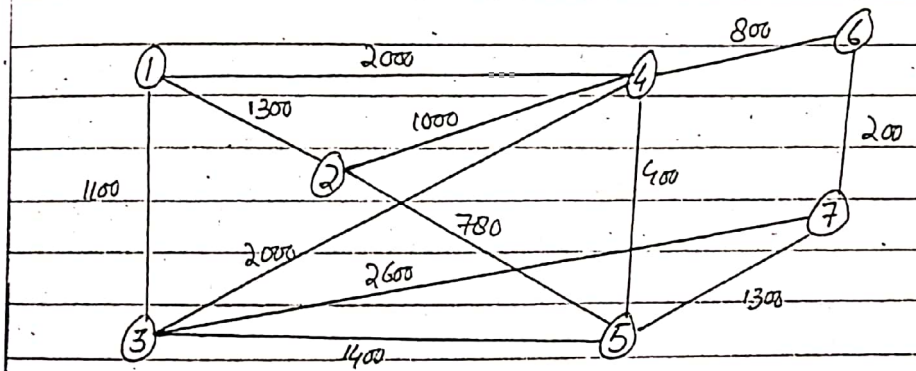
Q3:- Find the minimal spanning tree in the following network.



Sol:- ~~The minimal spanning~~

①	②	③
~~1 1300 2~~	~~2 1300 1~~	3 1100 1
1 1100 3	~~2 1000 4~~	3 2000 4
1 2000 4	~~2 780 5~~	3 1400 5
		3 2600 7

④	⑤	⑥
~~4 2000 1~~	5 780 2	~~6 800 4~~
~~4 1100 3~~	5 1400 3	~~6 200 7~~
4 2000 3	~~5 400 4~~	
~~4 400 5~~	5 1300 7	
4 800 6		

⑦	
7 2600 3	; ~~7 200 6~~
7 1300 5	

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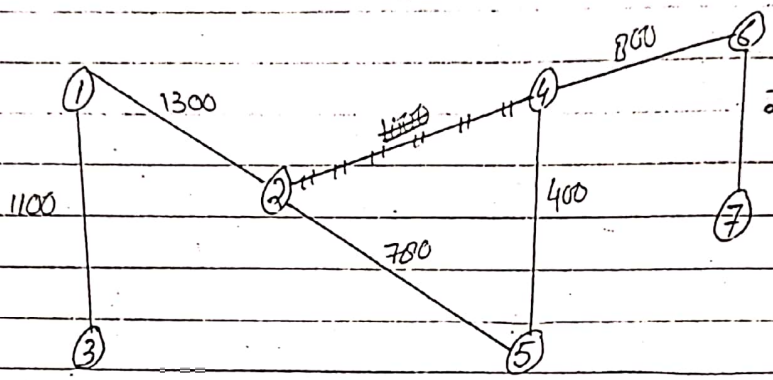
The minimal spanning is.



and the cable cost is.

$1100 + 1300 + 780 + 800 + 400 + 200 = 4580$

Imp

Ans:-

Shortest Route Problems:-

Acyclic Network:-

A network is called to be acyclic if there is no connection to a node to itself.

Algorithum For Finding Shortest Route In Cyclic Network:-

We shall use the following notation.

d_{ij} = Distance b/w adjecent node i and j.

U_j = Shortest distance from node 1 to node j

= minimum $[d_{ij} + U_i]$

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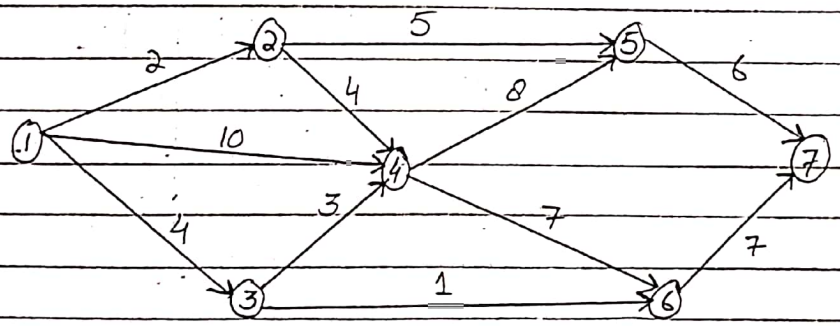
where, U_i is the distance of node i from node 1 and minimum is taken over all the nodes preceeding node j directly.

Q1: Find the shortest distance b/w node 1 and node 7 in the following network.



Sol:-

$\Rightarrow U_1 = 0$

$U_2 = d_{12} + U_1 = 2 + 0$

$\Rightarrow U_2 = 2$

$U_3 = d_{13} + U_1 = 4 + 0$

$\Rightarrow U_3 = 4$

$U_4 = \min\{d_{14}+U_1 ; d_{24}+U_2 ; d_{34}+U_3\}$.

$= \min\{10+0 ; 4+2 ; 3+4\}$.

$= \min\{10 ; 6 ; 7\}$

$\Rightarrow U_4 = 6$

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$U_5 = \min\{d_{25} + U_2 ; d_{45} + U_4\}$
$= \min\{5+2 ; 8+6\}$.
$= \min\{7, 14\}$.

$\Rightarrow U_5 = 7$.

$U_6 = \min\{d_{36} + U_3 ; d_{46} + U_4\}$
$= \min\{1+4 ; 7+6\}$.
$= \min\{5, 13\}$.

$\Rightarrow U_6 = 5$

and:

$U_7 = \min\{d_{57} + U_5 ; d_{67} + U_6\}$.
$= \min\{6+7 ; 7+5\}$.
$= \min\{13 ; 12\}$.

$\Rightarrow U_7 = 12$

Hence, shortest distance is 12 units and shortest route is:

1 → 3 → 6 → 7 Ans:-

Q2.11 Find the shortest distance from city 1 to city 8. in the following network.

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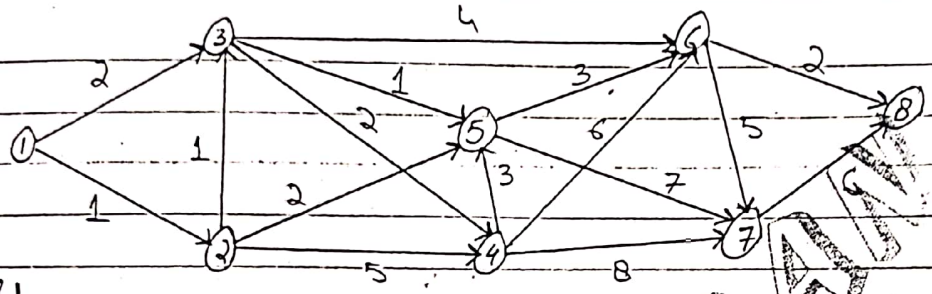

Sol:-

$\Rightarrow U_1 = 0.$

$U_2 = d_{12} + U_1 = 1 + 0.$

$\Rightarrow U_2 = 1$

$U_3 = \min\{d_{13} + U_1, d_{23} + U_2\}.$

$= \min\{2+0, 1+1\}.$

$= \min\{2, 2\}$

$\Rightarrow U_3 = 2.$

$U_4 = \min\{d_{24} + U_2, d_{34} + U_3\}.$

$= \min\{5+1, 2+2\}.$

$= \min\{6, 4\}.$

$\Rightarrow U_4 = 4$

$U_5 = \min\{d_{25} + U_2, d_{35} + U_3, d_{45} + U_4\}.$

$= \min\{2+1, 1+2, 3+4\}.$

$= \min\{3, 3, 7\}$

$\Rightarrow U_5 = 3$

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(20)

$U_6 = \min [d_{36} + U_3 ; d_{46} + U_4 ; d_{56} + U_5]$.
$= \min \{4+2 ; 6+4 ; 3+3\}$.
$= \min \{6 ; 10 ; 6\}$.

$\Rightarrow U_6 = 6$.

$U_7 = \min \{d_{47} + U_4 , d_{57} + U_5 , d_{67} + U_6\}$.
$= \min \{8+4 ; 7+3 ; 5+6\}$.
$= \min \{12 , 10 , 11\}$.

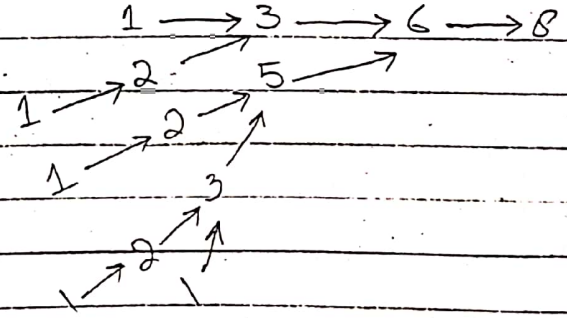
$U_7 = 10$.

and;

$U_8 = \min \{d_{68} + U_6 , d_{78} + U_7\}$.
$= \min \{2+6 ; 6+10\}$.
$= \min \{8 , 16\}$.

$\Rightarrow U_8 = 8$.

Hence, the shortest distance is 8 units and the shortest route is.



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i-e.

$1 \rightarrow 3 \rightarrow 6 \rightarrow 8$

or $1 \rightarrow 2 \rightarrow 3 \rightarrow 6 \rightarrow 8$

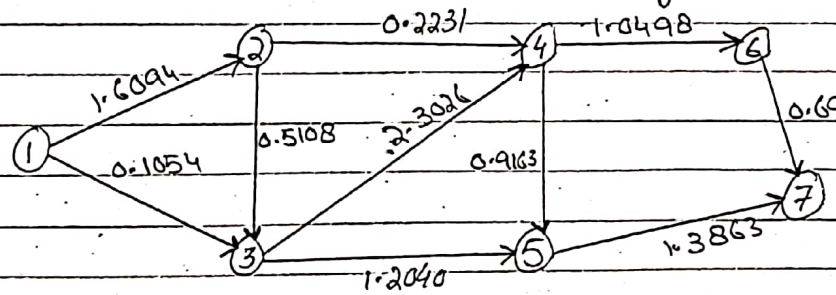
or $1 \rightarrow 2 \rightarrow 5 \rightarrow 6 \rightarrow 8$

or $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 8$

or $1 \rightarrow 3 \rightarrow 5 \rightarrow 6 \rightarrow 8$

Ans:-

Q3.11 Find the shortest distance and the shortest route in the following network.



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Sol:-

$\Rightarrow U_1 = 0$

$U_2 = d_{12} + U_1 = 1.6094 + 0$

$\Rightarrow U_2 = 1.6094$

$U_3 = \min \{d_{13} + U_1 ; d_{23} + U_2\}.$

$= \min \{0.1054 + 0 ; 0.5108 + 1.6094\}.$

$= \min \{0.1054, 2.1202\}.$

$\Rightarrow U_3 = 0.1054.$

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(20)

$U_4 = \min\{d_{24} + U_2 ; d_{34} + U_3\}$.

$= \min\{0.2231 + 1.6094 ; 2.3026 + 0.1054\}$.

$= \min\{1.8325 ; 2.408\}$.

$\Rightarrow U_4 = 1.8325$

$U_5 = \min\{d_{35} + U_3 ; d_{45} + U_4\}$.

$= \min\{1.2040 + 0.1054 ; 0.9163 + 1.8325\}$.

$= \min\{1.3094 ; 2.7488\}$.

$\Rightarrow U_5 = 1.3094$.

$U_6 = d_{46} + U_4$

$\Rightarrow U_6 = 1.0498 + 1.8325 = 2.8823$.

and

$U_7 = \min\{d_{57} + U_5 ; d_{67} + U_6\}$.

$= \min\{1.3863 + 1.3094 ; 0.6931 + 2.8823\}$

$= \min\{2.6957 , 3.5754\}$.

$\Rightarrow U_7 = 2.6957$

Hence, the shortest distance is 2.6957 and the shortest route is.

1 → 3 → 5 → 7 Ans:-

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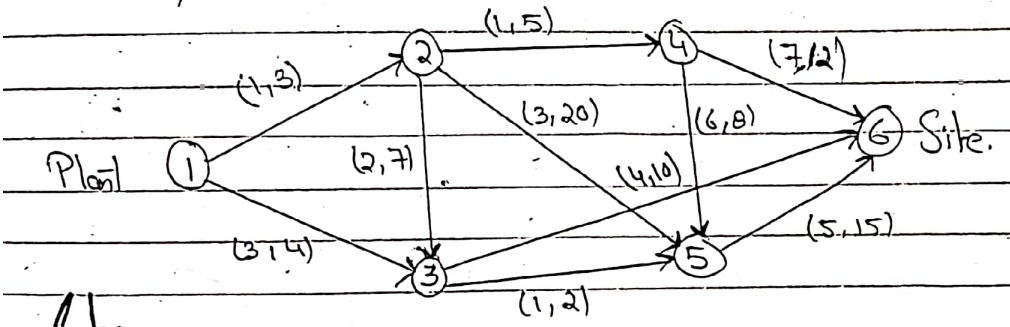
Q4:- A truck must delivered concreel from a plant to a construction site. The network in the fig. below represents the available routes blw the plant and site. Each route is designed with two pieces of informations (d, t) where d is the length of the route and t is the time, the trucks takes to cross the road segement. What is the best route from point to construction site.



Sol:-

First we find the shortest distance from plant to construction site.

$\Rightarrow U_1 = 0$

$U_2 = d_{12} + U_1 = 1 + 0$

$\Rightarrow U_2 = 1$

$U_3 = \min \{ d_{13} + U_1, d_{23} + U_2 \}.$

$= \min \{ 3 + 0 ; 2 + 1 \}.$

$= \min \{ 3, 3 \}.$

$\Rightarrow U_3 = 3$

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$U_4 = d_{24} + U_2 = 1 + 1$

$\Rightarrow U_4 = 2.$

$U_5 = \min\{d_{25} + U_2 , d_{35} + U_3 , d_{45} + U_4\}.$

$= \min\{3 + 1 ; 1 + 3 , 6 + 2\}.$

$= \min\{4, 4, 8\}.$

$\Rightarrow U_5 = 4$

and

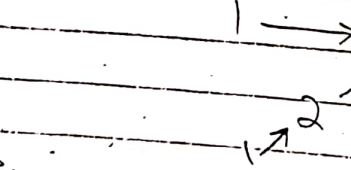
$U_6 = \min\{d_{36} + U_3 , d_{46} + U_4 , d_{56} + U_5\}.$

$= \min\{4 + 3 ; 7 + 2 , 5 + 4\}.$

$= \min\{7, 9, 9\}.$

$\Rightarrow U_6 = 7$

So, the shortest distance is 7 and the shortest route is.



i-e.

$1 \longrightarrow 3 \longrightarrow 6$

or $1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 6.$

Next; we find the shortest time from plant to the construction site.

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$\Rightarrow T_1 = 0$

$T_2 = t_{12} + T_1 = 3 + 0$

$\Rightarrow T_2 = 3$

$T_3 = \min \{t_{13} + T_1 , t_{23} + T_2\}.$

$= \min \{4 + 0 ; 7 + 3\}.$

$T_3 = \min \{4, 10\}.$

$\Rightarrow T_3 = 4$

$T_4 = t_{24} + T_2 = 5 + 3$

$\Rightarrow T_4 = 8$

$T_5 = \min \{t_{25} + T_2 , t_{35} + T_3 , t_{45} + T_4\}.$

$= \min \{20 + 3 ; 2 + 4 ; 8 + 8\}.$

$= \min \{23, 6, 16\}.$

$\Rightarrow T_5 = 6.$

and

$T_6 = \min \{t_{36} + T_3 , t_{46} + T_4 , t_{56} + T_5\}.$

$= \min \{10 + 4 , 12 + 8 , 15 + 6\}.$

$= \min \{14, 20, 21\}.$

$\Rightarrow T_6 = 14$

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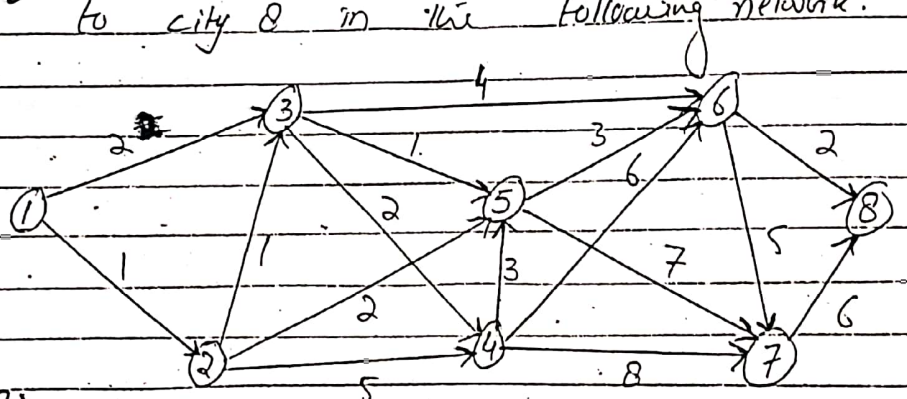
Hence, the minimum time is 14 units and the path is given as.

$$1 \longrightarrow 3 \longrightarrow 6$$

Hence, best route is

$$1 \longrightarrow 3 \longrightarrow 6$$ Ans:-

Q5.// Find the shortest route from city 4 to city 8 in the following network.



Sol:-

$$u_4 = 0$$

$$u_5 = d_{45} + u_4 = 3 + 0$$

$$\Rightarrow u_5 = 3$$

$$u_6 = \min\{d_{46} + u_4 , d_{56} + u_5\}$$
$$= \min\{6 + 0 ; 3 + 3\}$$
$$= \min\{6, 6\}$$

$$\Rightarrow u_6 = 6$$

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$U_7 = \min\{d_{47} + U_4,\ d_{57} + U_5,\ d_{67} + U_6\}$.

$= \min\{0 + 0,\ 7 + 3,\ 5 + 6\}$.

$= \min\{8,\ 10,\ 11\}$.

$\Rightarrow U_7 = 8$

and

$U_8 = \min\{d_{68} + U_6,\ d_{78} + U_7\}$.

$= \min\{2 + 6,\ 6 + 8\}$.

$= \min\{8,\ 14\}$.

$\Rightarrow U_8 = 8$

So, the minimum distance is 8 units and shortest route is

$4 \longrightarrow 6 \longrightarrow 8$

$4 \nearrow 5 \nearrow$

i.e. $4 \longrightarrow 6 \longrightarrow 8$

$4 \longrightarrow 5 \longrightarrow 6 \longrightarrow 8$. Ans:-

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Shortest Route Problem For Cyclic Network:-

Cyclic Network:-

The network in which we can flow in both the directions is called cyclic network.

Procedure For Finding Cyclic Network Problem:-

To solve the cyclic network problem we convert the given problem in assignment problem

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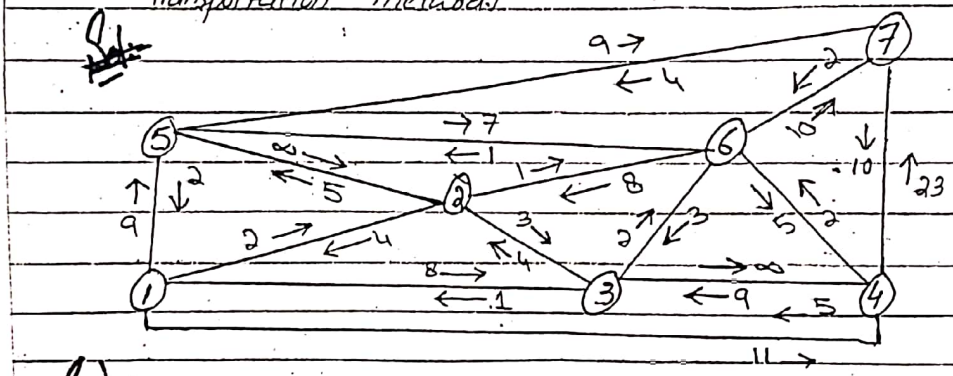
and solve by using the assignment method.

Q1. Solve the following shortest distance problem. Find the distance from route 1 to 7.

OR

Solve the above problem using transportation technique or by transship method.
(∵ Assignment method is special case of the transportation method)

Sol:-



Sol:-

First we convert the problem in assignment model.

	2	3	4	5	6	7
1	2	8	11	9	x	x
2	0	3	x	5	1	x
3	4	0	x	x	2	x
4	x	9	0	x	2	23
5	x	x	x	0	7	9
6	8	3	5	1	0	10

(Here, x is stands for infinity)

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Next, we solve the following problem by Assignment method.

Step I:-

Subtract the smallest element of each row from the corresponding row.

0	6	9	7	x	x	$P_1 = 2$
0	3	x	5	1	x	$P_2 = 0$
4	0	x	x	2	x	$P_3 = 0$
x	9	0	x	2	23	$P_4 = 0$
x	x	x	0	7	9	$P_5 = 0$
8	3	5	1	0	10	$P_6 = 0$

Step II :-

Subtract the smallest element of each coulumn from the corresponding coulumn.

0	6	9	7	x	x
0	3	x	5	1	x
4	0	x	x	2	x
x	9	0	x	2	14
x	x	x	0	7	0
8	3	5	1	0	1

$q_1 = 0$ $q_2 = 0$ $q_3 = 0$ $q_4 = 0$ $q_5 = 0$ $q_6 = 9$

Step III :-

Draw the min. no. of horizental and vertical lines to cross all the zeros.

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0	6	8	x	1	x
0	3	x	5	1	x
1	0	x	x	2	x
x	1	0	x	2	14
x	x	0	7	1	0
3	5	1	0	1	

Since;

Min. no. of lines drawn < Order of original matrix.

The minimum uncovered element is 1. Subtract this element from the uncovered elements; add to the elements in the intersection of the lines drawn and leave the remaining unchanged. Then again draw the minimum no. of the lines to cross all the zeros.

0	5	8	6	x	x
0	2	x	4	0	x
5	0	x	x	2	x
x	9	0	x	3	14
x	x	x	0	1	0
9	3	5	1	0	1

Since;

Min. no. of the lines drawn < Order of original matrix

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The minimum no. of the uncovered elements is 1. Subtract this element from uncovered elements add to the elements in the intersection of the lines and leave the remaining unchange. Then again draw the min. no. of lines to cross all the zeros.

0	4	7	5	x	x
0	1	x	3	0	x
6	0	x	x	3	x
x	9	0	x	3	14
x	x	x	0	8	0
9	~~2~~	~~4~~	0	0	0

Now;

Min. No. of lines drawn = Order of original matrix

Step V:-

	2	3	4	5	6	7
1	[0]	4	7	5	x	x
2	~~X~~	1	x	3	[0]	x
3	6	[0]	x	x	3	x
4	x	9	[0]	x	3	14
5	x	x	x	[0]	8	(0)
6	9	2	4	(0)	~~X~~	[0]

The shortest distance from 1 to 7 node.

$$\Sigma p's + \Sigma q's + 1 + 1$$
$$= 2 + 9 + 1 + 1$$
$$= 13$$

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and the shortest route.

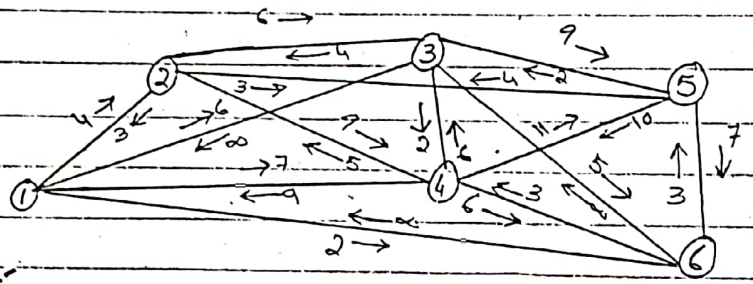
$1 \rightarrow 2 \rightarrow 6 \rightarrow 7$

or

$1 \rightarrow 2 \rightarrow 6 \rightarrow 5 \rightarrow 7$.

Ans:-

Q2.11 Find the shortest route from node 1 to node 5. in the following network.



Sol:-

First we convert the problem in the assignment model.

	2	3	4	5	6
1	4	6	7	x	2
2	0	6	9	3	x
3	4	0	2	9	5
4	5	6	0	11	6
6	x	x	3	3	0

Now, we solve the problem by using assignment method.

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Step I:-

Subtract the smallest element of each row from the corresponding row.

2	4	5	x	0	$P_1 = 2$
0	6	9	3	x	$P_2 = 0$
4	0	2	9	5	$P_3 = 0$
5	6	0	11	6	$P_4 = 0$
x	x	3	3	0	$P_5 = 0$

Step II:-

Subtract the smallest element of each coulumn from the corresponding coulumn.

2	4	5	x	0
0	6	9	0	x
4	0	2	6	5
5	6	0	8	6
x	x	3	0	0

$q_1 = 0$ $q_2 = 0$ $q_3 = 0$ $q_4 = 3$ $q_5 = 0$

Step III:-

Draw the min. no. of horizental and vertical lines to cross all the zeros.

2	4	5	x	0
0	6	9	0	x
4	0	2	6	5
5	6	0	8	6
x	x	3	0	0

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Since,

Min. no. of lines drawn = Order of original matrix

Step IV:-

Complete the assignment.

	2	3	4	5	6
1	2	4	5	x	[0]
2	[0]	6	9	~~0~~	x
3	4	[0]	2	6	5
4	5	6	[0]	0	6
6	x	x	3	[0]	~~0~~

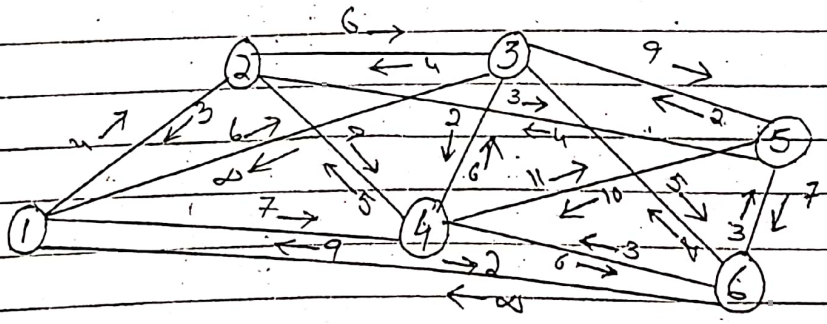
Assignment is completed. So shortest distance is

$$\Sigma p's + \Sigma q's = 2 + 3$$
$$= 5$$

and shortest route is:

$$1 \longrightarrow 6 \longrightarrow 5$$

Q3. Find the shortest route from node 3 to node 6 in the following network.



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Sol:- First, we convert the problem in the assignment model.

	1	2	4	5	6
1	0	4	7	x	2
2	3	0	9	3	x
3	x	4	8	9	5
4	9	5	0	11	6
5	x	4	10	0	7

Step I:-

Subtract the smallest element of each row from the corresponding row.

0	4	7	x	2	$P_1 = 0$
3	0	9	3	x	$P_2 = 0$
x	2	0	7	3	$P_3 = 2$
9	5	0	11	6	$P_4 = 0$
x	4	10	0	7	$P_5 = 0$

Step II:-

Subtract the smallest element of each column from the corresponding column.

0	4	7	x	0
3	0	9	3	x
x	2	0	7	1
9	5	0	11	4
x	4	10	0	5

$q_{v_1=0}$ $q_{v_2=0}$ $q_{v_3=0}$ $q_{v_4=0}$ $q_{v_5=2}$

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Step III :-

Draw the min. no. of horizental ~~lines~~ and vertical lines to cross all zeros.

0	4	7	x	0
3	0	4	3	x
x	2	0	7	1
9	5	0	11	1
x	4	10	0	5

Since:

Min. no. of lines drawn < Order of orginal matrix

The min uncovered element is "1". Subtract this element from the ~~union~~ uncovered elements, add to the ~~lines~~ elements in the intersection of lines and leave the remaining unchanged. Then again draw the min. no. of horizental and vertical lines to cross all zeros.

0	4	6	x	0
3	0	0	3	x
x	1	0	6	0
8	4	0	10	3
x	4	4	0	5

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Step V :-

Complete the assignment

	1	2	4	5	6
1	[0]	4	8	x	~~0~~
2	3	[0]	10	3	x
3	x	1	~~0~~	6	[0]
4	8	4	[0]	10	3
5	x	4	11	[0]	5

Assignment is completed. So shortest distance is

$$\Sigma p's + \Sigma q's + 1 = 2 + 2 + 1$$
$$= 5$$

and shortest route is.

~~1~~ 3 → 6.

Ans :-

Maximal Flow Problem :-

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One Way Problem :-

In the maximal flow problem, a source node and a destination node are linked together through network of one-way arcs. Each arc has a maximum amount flow capacity. The objective is to find the max amount of flow between source and destination nodes.

[Note: The processes of finding successive approximation is called iteration.]

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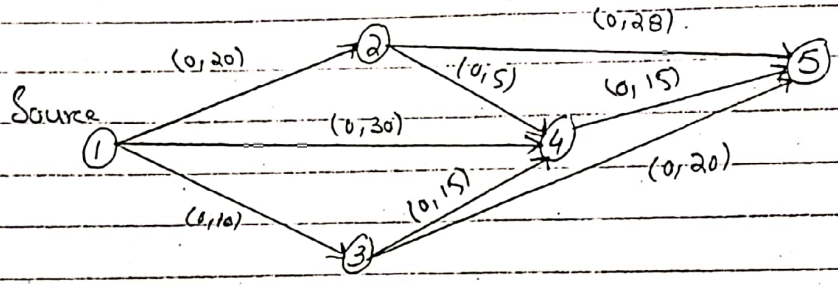
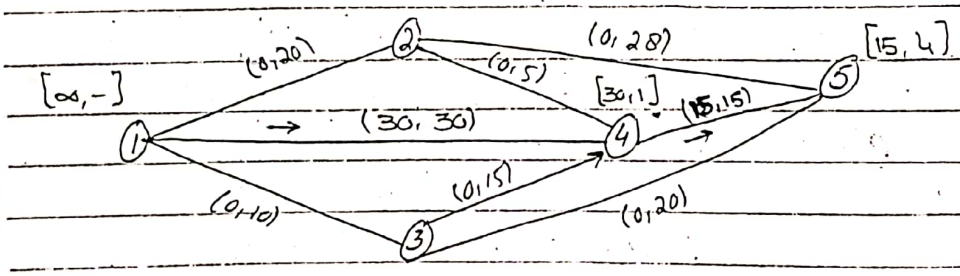
Q:-1 Determine the maximal flow b/w the node 1 and node 5 in the following network



Sol:-

1st Iteration:-



Path is

$1 \longrightarrow 4 \longrightarrow 5$

$\text{flow} = f_1 = \min\{\infty, 30, 15\}$

$\Rightarrow f_1 = 15.$

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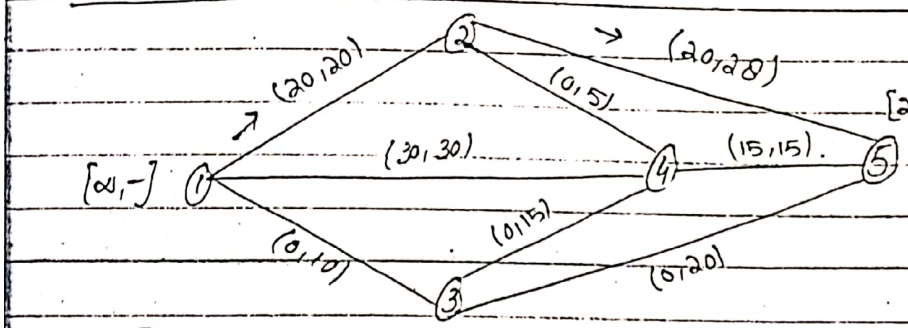
2nd Iteration:



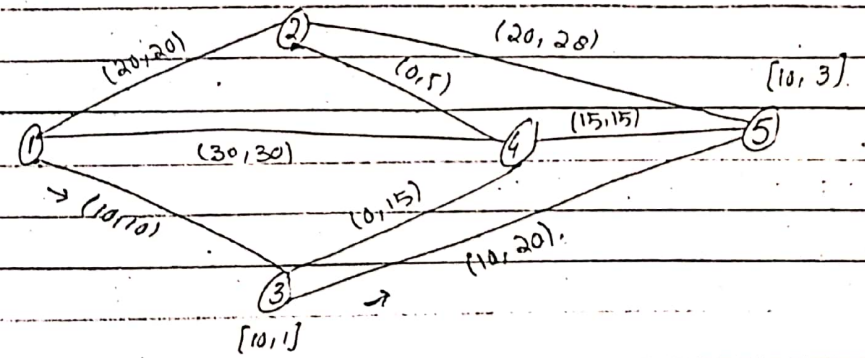
Path is

$1 \rightarrow 2 \rightarrow 5$

and flow = $f_2 = \min\{\infty, 20, 20\}$

$\Rightarrow f_2 = 20.$

3rd Iteration:



Path is

$1 \rightarrow 3 \rightarrow 5$

and flow = $f_3 = \min\{\infty, 10, 10\}$

$\Rightarrow f_3 = 10.$

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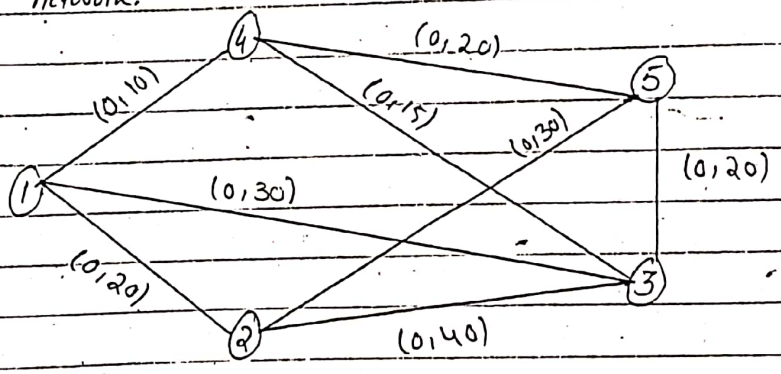
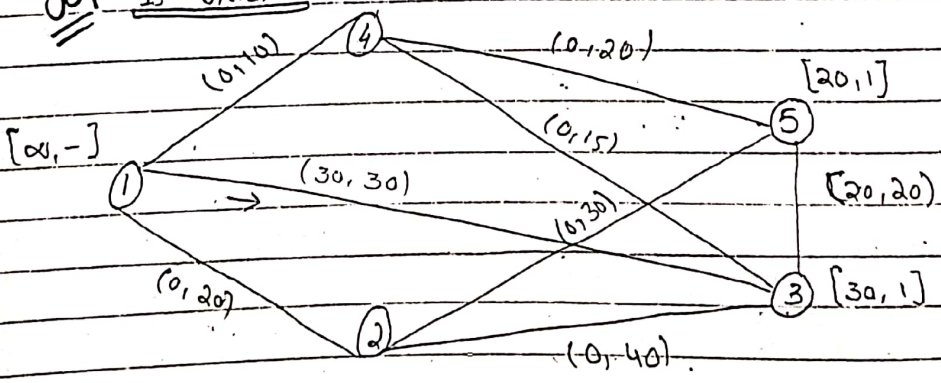
No, further brake through is possible. So maximal flow is

$$f = f_1 + f_2 + f_3$$
$$f = 15 + 20 + 10$$
$$f = 45 \quad \text{Ans:-}$$

Q2. Find the maximal flow b/w node 1 and node 5 in the following network.



Sol:- 1st Iteration



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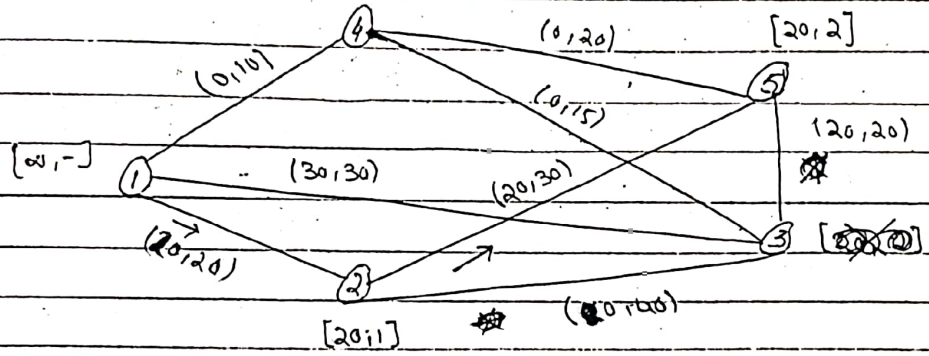
Path is

$1 \rightarrow 3 \rightarrow 5$

and flow = $f_1 = \min\{\infty, 30, 20\}$.

$\Rightarrow f = 20$.

2nd Iteration:



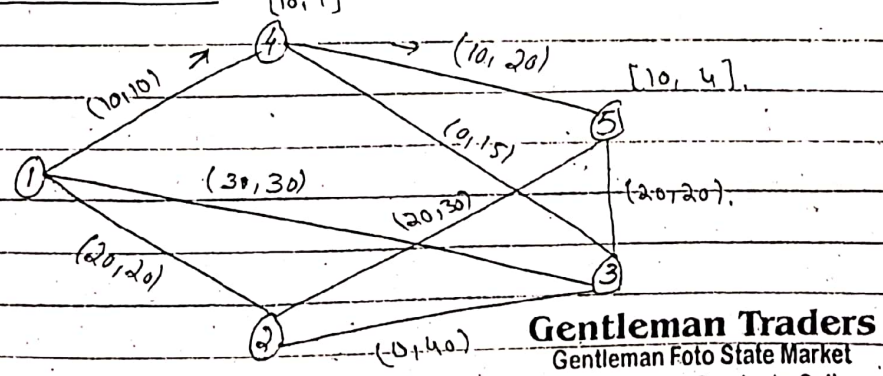
Path is

$1 \rightarrow 2 \rightarrow 5$

and flow = $f_2 = \min\{\infty, 20, 20\}$.

$\Rightarrow f_2 = 20$.

3rd Iteration:



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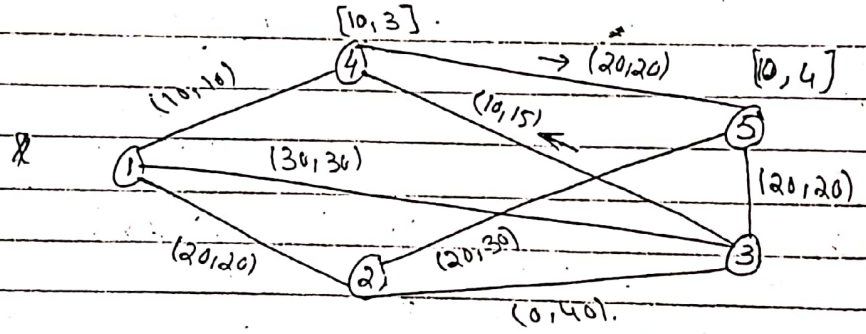
Path is

$1 \rightarrow 4 \rightarrow 5$

and flow $= f_3 = \min\{\infty, 10, 10\}$.

$\Rightarrow f_3 = 10$.

4th Iteration:



Path is:

$3 \rightarrow 4 \rightarrow 5$

and flow $= \min\{\infty, 10, 10\}$.

$f_4 = 10$.

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Since 10 unit flow is already available at node 3 which is moved from node 3 to 5 through 4. No further brake through is available. Therefore maximal flow is

$f = f_1 + f_2 + f_3 + f_4$

$= 20 + 20 + 10 + 10$

$f = 60$ Ans:-

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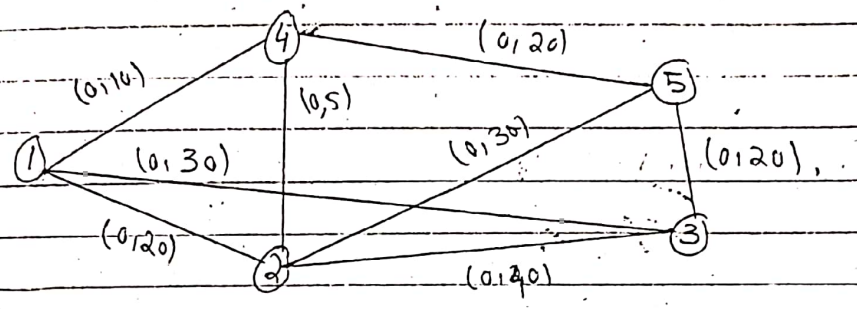
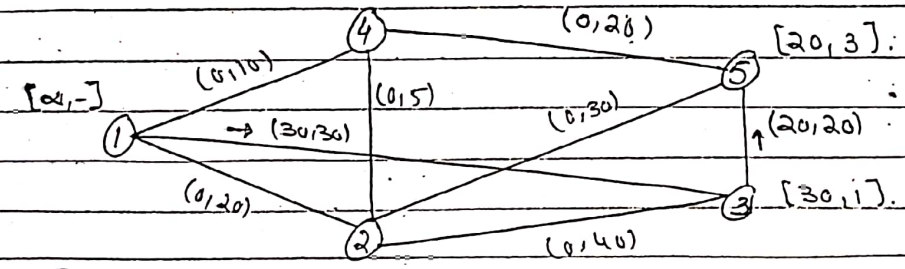
(Assignment 1) (226)

Q3:- Determine the maximal flow in the following network from node 1 to 5.



Sol:-

1st Iteration:



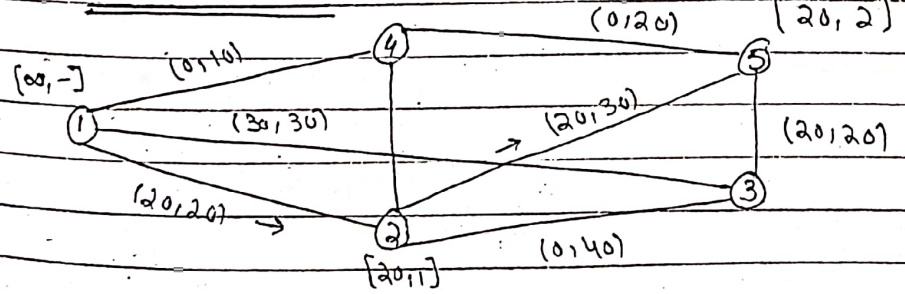
Path is

$1 \longrightarrow 3 \longrightarrow 5$

and flow $= f_1 = \min\{\infty, 30, 20\}$

$\Rightarrow f_1 = 20.$

2nd Iteration:



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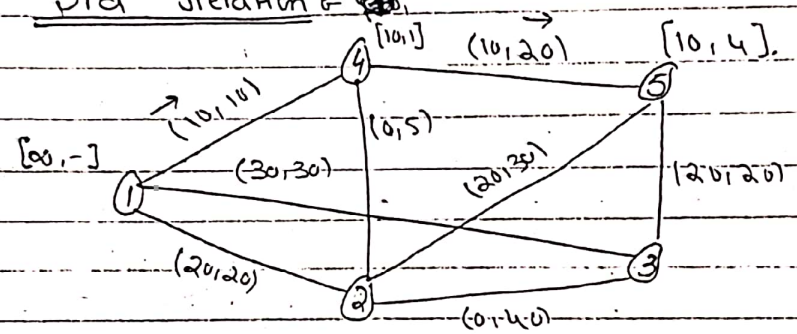
Path is.

$1 \rightarrow 2 \rightarrow 5$

and flow = f_2 = min $\{\infty, 20, 20\}$.

$\Rightarrow f_2 = 20.$

3rd Iteration:-



Path is

$1 \rightarrow 4 \rightarrow 5$

and ~~path~~ flow = f_3 = min $\{\infty, 10, 10\}$.

$\Rightarrow f_3 = 10.$

Hence,

the maximal flow is.

$f = f_1 + f_2 + f_3$

$= 20 + 20 + 10.$

$f = 50.$ Ans:-

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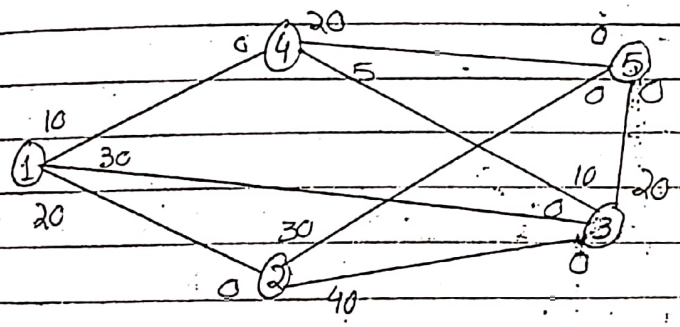
Maximal Flow Problem:-

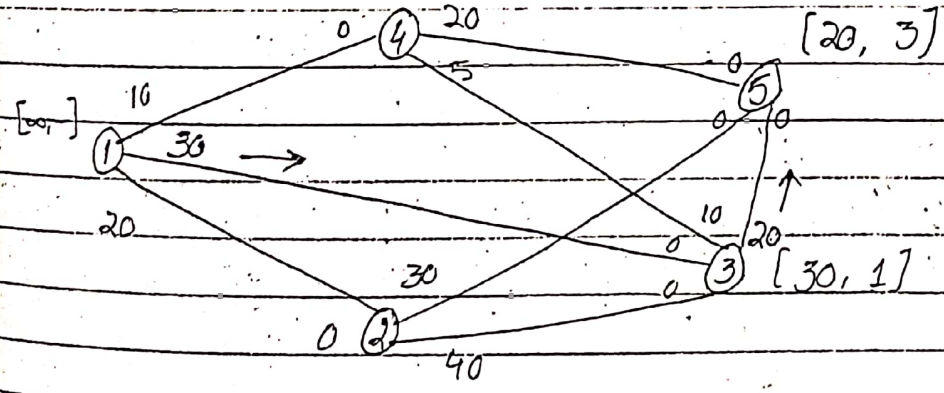
Two-Way Problem:-

Q1. Determine the maximal flow in the following network from node 1 to node 5. The min and max. capacities are shown below.



Sol:-

1st Iteration



Path is

$1 \rightarrow 3 \rightarrow 5$

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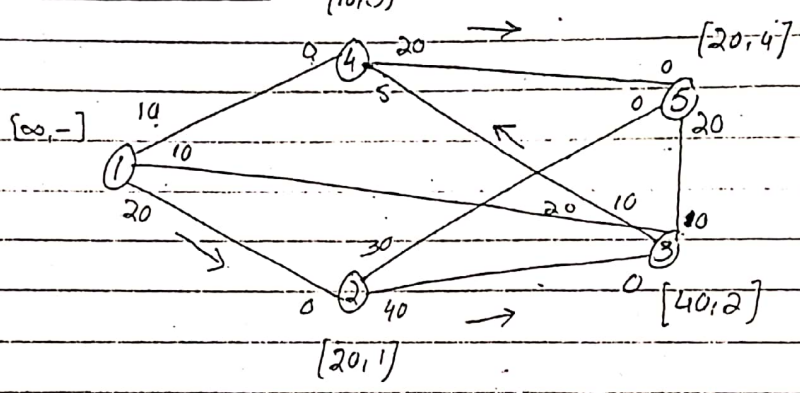
and Flow = f_1 = min {∞, 30, 20}.

f_1 = 20.

2nd Iteration:



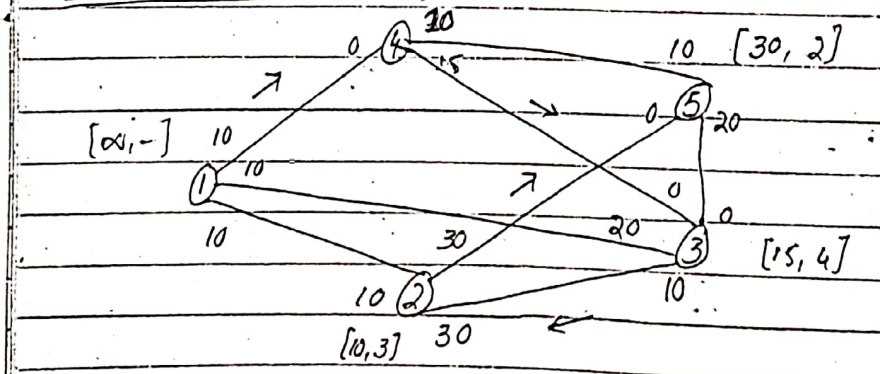
Path is

1 → 2 → 3 → 4 → 5

and Flow = f_2 = min {∞, 20, 40, 10, 20}

⇒ f_2 = 10

3rd Iteration:



Path is 1 → 4 → 3 → 2 → 5.

and Flow = f_3 = min {∞, 10, 15, 10, 30}

⇒ f_3 = 10

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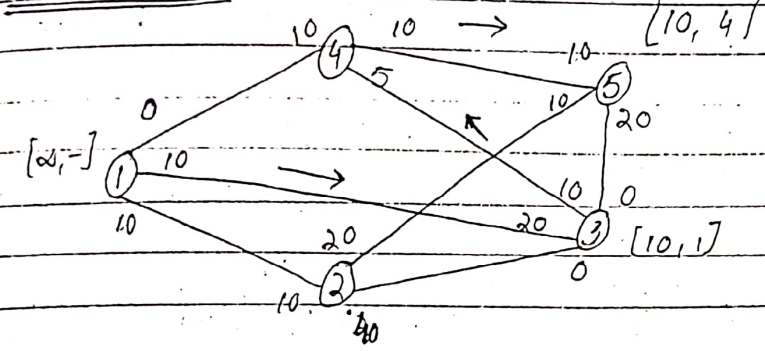
Q3.a)

4th Iteration: [10, 3]

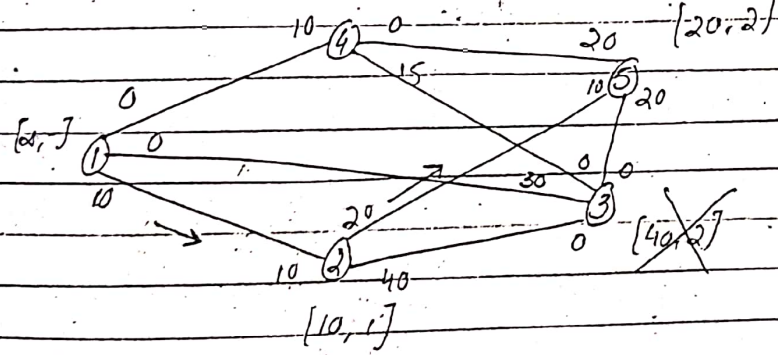


Path is $1 \rightarrow 3 \rightarrow 4 \rightarrow 5$

and Flow = f_4 = min {∞, 10, 10, 10}.

$\Rightarrow f_4 = 10.$

5th Iteration:



Path is $1 \rightarrow 2 \rightarrow 5$

and Flow = f_5 = min {∞, 10, 20}.

$\Rightarrow f_5 = 10.$

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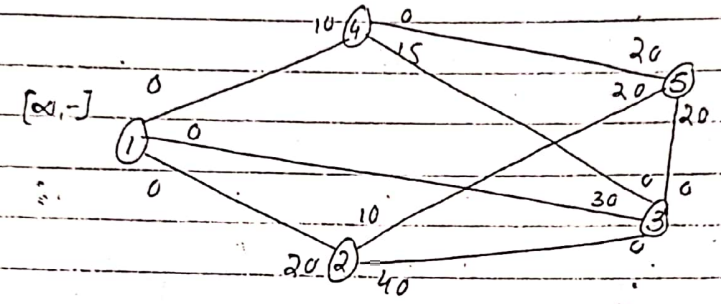
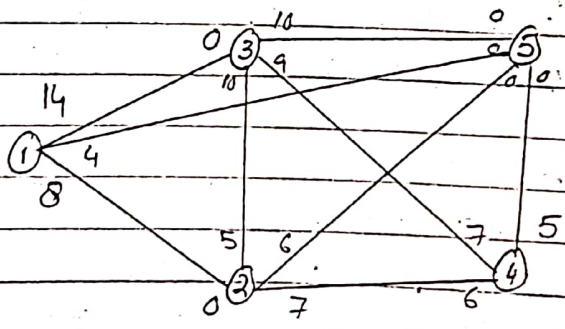
6th Iteration:



Since no further brake through is possible.
Therefore, max. flow is

$$f = f_1 + f_2 + f_3 + f_4 + f_5$$
$$= 20 + 10 + 10 + 10 + 10$$
$$f = 60 \quad \text{Ans:-}$$

Q2.n Determine the maximal flow b/w node 1 and node 5. From the following network.



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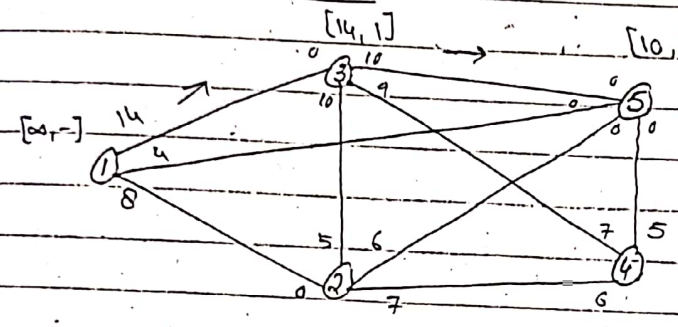
Sol:-

1st Iteration:-

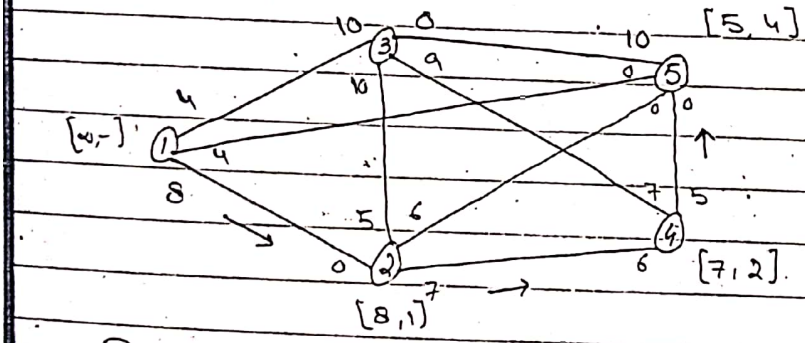


Path is $1 \rightarrow 3 \rightarrow 5$

and flow = $f_1 = \min\{\infty, 14, 10\}$.

$\Rightarrow f_1 = 10$

2nd Iteration:-



Path is

$1 \rightarrow 2 \rightarrow 4 \rightarrow 5$

and flow = $f_2 = \min\{\infty, 8, 7, 5\}$.

$\Rightarrow f_2 = 5$

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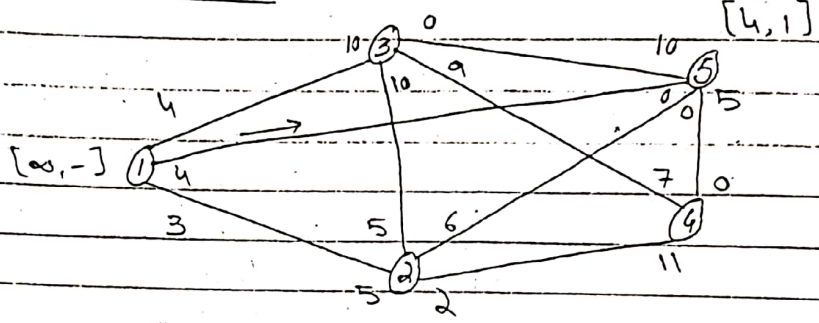
3rd Iteration :



Path is $1 \rightarrow 5$

and flow = $f_3 = 4$.

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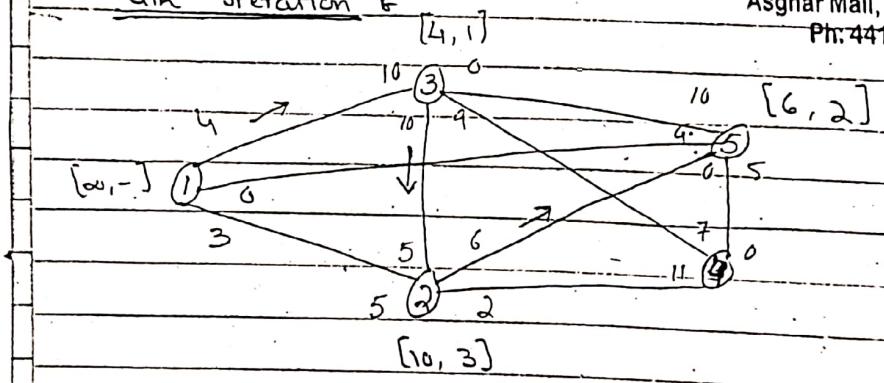
4th Iteration :



Path is

$1 \rightarrow 3 \rightarrow 2 \rightarrow 5$

flow = $f_4 = \min[\infty, 4, 10, 6]$

$\Rightarrow f_4 = 4$

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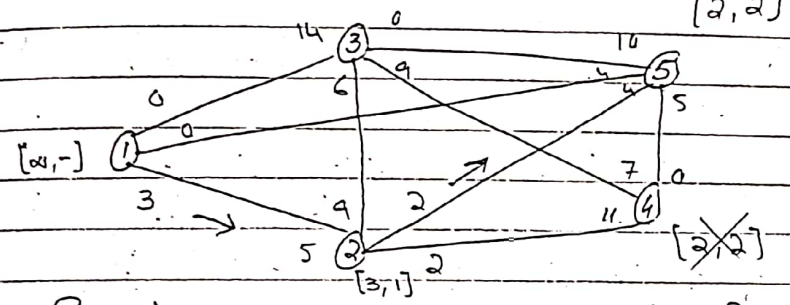
5th Iteration:

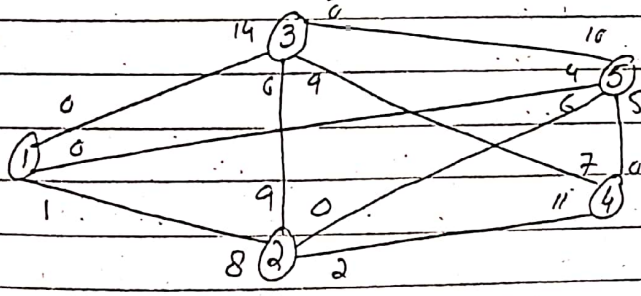


Path is ~~1 → 3 → 2 → 5~~ 1 → 2 → 5

and flow = f_5 = min {∞, ~~4~~ 3, 2, ~~10~~, 6}

$\Rightarrow f_5 = 2$

6th Iteration:



Since, no further brake through is possible.

Therefore maximal flow is

$f = f_1 + f_2 + f_3 + f_4 + f_5$

$= 10 + 5 + 4 + 4 + 2$

$f = 25$ Ans:

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Transhipment Model:-

Q:- Write a note on transhipment model.

Sol:-

In some transportation problems, we consider the possibility shipping a commodity through intermediate nodes before reaching the final destination. This case is referred as transhipment model.

In the transhipment model, the supply and the demand node of the model are divided into

i) Pure Supply nodes
ii) Pure demand nodes.
iii) Transhipment node.

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The amount ~~of~~ of supply and demand at these nodes are computed as.

Supply at Pure (supply) Nodes = Original Supply.

Demand at Pure demand Nodes = Original Demand.

Supply at a transhipment Node = Original Supply + Buffer.

Demand at a transhipment Node = Original Demand + Buffer

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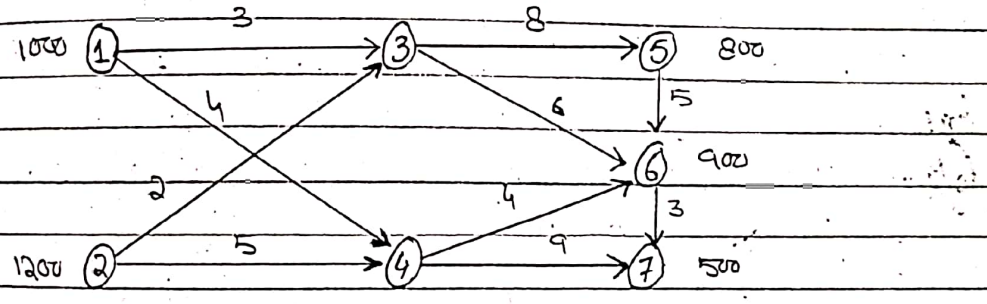
The buffer amount should be sufficiently large to allow the entire original supply or demand to pass through the transhipment node and it is usually equals to total supply (or total demand).

Q:-1:- Write the ~~following~~ transhipment model of the following problem.



Sol:-

Pure Supply Nodes = 1, 2.
Pure Demand Nodes = 7
Transhipment Nodes = 3, 4, 5, 6
Buffer Stock = 2200 (Total Supply)

The corresponding transhipment model is

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Source	Destination 3	4	5	6	7	Supply
1	3	4	M	M	M	1000
2	6	5	M	M	M	1200
3	0	M	0	6	M	2200
4	M	0	M	4	9	2200
5	M	M	0	5	M	2200
6	M	M	M	0	3	2200
Demand	2200	2200	3000	3100	500	

which is required transshipment model.

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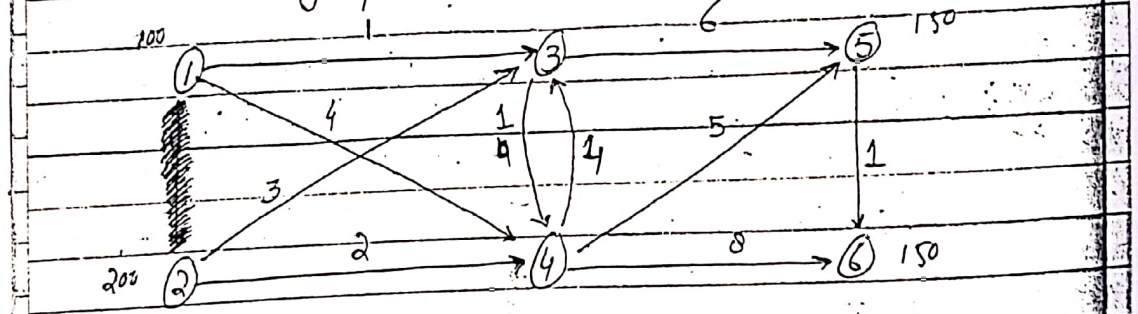
Q2- Solve the transshipment model of the following problem.



Sol:-

First, we write the transshipment model of the above problem.

Pure Supply Node = 1, 2

Pure Demand Node = 6

Transshipment Nodes = 3, 4, 5

Buffer Stock = 300.

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Source	Destination 3	4	5	6	Supply
1	1	4	M	M	100
2	3	2	M	M	200
3	0	1	6	M	300
4	4	0	5	8	300
5	M	M	0	1	300
Demand	~~450~~ 300	~~450~~ 300	450	150	

Next, we solve the model starting by NWC - method.

	100 1	4	M	M	~~100~~
	200 3	2	M	M	~~200~~
	0 0	300 1	6	M	~~300~~
	4	0 0	300 5	8	~~300~~
	M	M	150 0	150 1	~~300~~ ~~150~~

$\emptyset$ ~~200~~ ~~300~~ $\emptyset$ ~~300~~ ~~450~~ ~~150~~ ~~150~~

Transportation Cost is:

$100 \times 1 = 100$

$200 \times 3 = 600$

$0 \times 0 = 0$

$300 \times 1 = 300$

$0 \times 0 = 0$

$300 \times 5 = 1500$

$150 \times 0 = 0$

$150 \times 1 = 150$

$= 2650$

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Now, we apply the UV-method to improve this solution. So assign a multiplier to each row and column.

	$V_1=1$	$V_2=2$	$V_3=7$	$V_4=8$
$U_1=0$	100 [1]	-2 [4]	-M [M]	-M [M]
$U_2=2$	⊕200 [3]	2 ⊕ [2]	-M [M]	-M [M]
$U_3=-1$	⊕0 [0]	300 ⊖ [1]	0 [6]	-M [M]
$U_4=-2$	-5 [4]	0 [0]	300 [5]	-2 [8]
$U_5=-7$	-M [M]	-M [M]	150 [0]	150 [1]

The minimum value in the -ive boxes is 200. Subtract this value from -ive box and add to the +ive boxes and leave the remaining unchanged. Then again assign the multiplier to each row and column.

	$V_1=1$	$V_2=2$	$V_3=7$	$V_4=8$
$U_1=0$	100 [1]	-2 [4]	-M [M]	-M [M]
$U_2=0$	-1 [3]	200 [2]	-M [M]	-M [M]
$U_3=-1$	200 [0]	100 [1]	0 [6]	-M [M]
$U_4=-2$	-5 [4]	0 [0]	300 [5]	-2 [8]
$U_5=-7$	-M [M]	-M [M]	150 [0]	150 [1]

Since, all the shadow costs are either zero or -ive. So, this is the optimal table and the optimal sol. is:

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$100 \times 1 = 100$

$200 \times 2 = 400$

$200 \times 0 = 0$

$100 \times 1 = 100$

$0 \times 0 = 0$

$300 \times 5 = 1500$

$150 \times 0 = 0$

$150 \times 1 = 150$

$= 2250$ Ans:-

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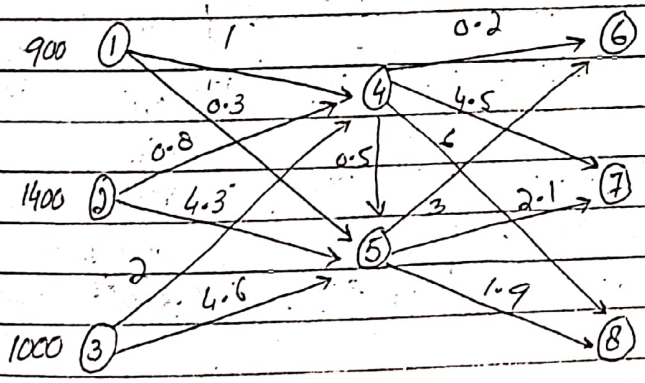
Q3* The network in the fig below shows the routes for shipping the cars from 3 plants to 3 dealers by the way of the two distributors. The shipping cost per car is shown in figure. Find the optimal solution of this problem that minimize the shipping cost.



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Sol:-

First we write the transhipment model of the problem.

Pure Supply Node = 1, 2, 3.

Pure Demand Node = 6, 7, 8

Transhipment Node = 4, 5

Buffer Stock = 3300

The corresponding transhipment model is.

Source \ Destination	4	5	6	7	8	Supply
1	1	0.3	M	M	M	900
2	0.8	4.3	M	M	M	1400
3	2	4.6	M	M	M	1000
4	0	0.5	0.2	4.5	6	3300
5	M	0	3	2.1	1.9	3300
Demand	3300	3300	1100	1000	1200	

Next, we solve the model by finding the starting solution with NWC-method.

	900 1	0.3	M	M	M	~~400~~
	1400 0.8	4.3	M	M	M	~~1400~~
	1000 2	4.6	M	M	M	~~1000~~
	0 0	3300 0.5	0.2	4.5	6	~~3300~~ ~~1200~~
	M	0 0	1100 3	1000 2.1	1200 1.9	~~3300~~ ~~2200~~
	~~1000~~	~~2400~~ 3300	~~3300~~	~~1100~~	~~1000~~	~~1200~~

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Transportation Cost is.

$900 \times 1 = 900$

$1400 \times 0.8 = 1120$

$1000 \times 2 = 2000$

$0 \times 0 = 0$

$3300 \times 0.5 = 1650$

$0 \times 0 = 0$

$1100 \times 3 = 3300$

$1000 \times 2.1 = 2100$

$1200 \times 1.9 = 2280$

$= 13350.$

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Now, we improve the sol. by using UV-method So, assign a multiplier to each row and coulumn.

	$V_1 = 1$	$V_2 = 1.5$	$V_3 = 4.5$	$V_4 = 3.6$	$V_5 = 3.4$
$U_1 = 0$	900 [1]	1.2 [0.3]	-M [M]	-M [M]	-M [M]
$U_2 = -0.2$	1400 [0.8]	-3 [4.3]	-M [M]	-M [M]	-M [M]
$U_3 = 1$	1000 [2]	-2.1 [4.6]	-M [M]	-M [M]	-M [M]
$U_4 = -1$	0 [6]	3300 ⊖ [0.5]	3.3 ⊕ [0.2]	-1.9 [4.5]	-3.6 [6]
$U_5 = -1.5$	-M [M]	0 ⊕ [6]	1100 ⊖ [3]	1000 [2.1]	1200 [1.9]

The minimum value in the -ive boxes is 1100. Subtract these value from -ive box, add to the +ive boxes and leave the remaining unchanged. Then again assign multipliers to each row and coulumn.

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$V_1 = 1$ $V_2 = 1.5$ $V_3 = 1.2$ $V_4 = 3.6$ $V_5 = 3.4$

$U_1 = 0$	900 0	1.2 ⊕ 0.3	M −M	M −M	M −M
$U_2 = -0.2$	1400 0.8	−3 4.3	M −M	M −M	M −M
$U_3 = 1$	1000 2	−2.1 4.6	M −M	M −M	M −M
$U_4 = -1$	0 ⊕ 0	2200 ⊖ 0.5	1100 0.2	3.9 4.5	−3.6 6
$U_5 = -1.5$	M −M	1100 0	3 −3.3	1000 2.1	1200 1.9

The minimum value in the -ive boxes is 900. Subtract this value from -ive boxes ~~and~~ added to the +ive boxes and leave the remaining unchanged. Then again assign the multipliers to each row and column.

$V_1 = -0.2$ $V_2 = 0.3$ $V_3 = 0.0$ $V_4 = 2.4$ $V_5 = 2.2$

$U_1 = 0$	-1.2 1	900 0.3	M −M	M −M	M −M
$U_2 = 1$	1400 0.8	−3 4.3	M −M	M −M	M −M
$U_3 = 2.2$	1000 2	−2.1 4.6	M −M	M −M	M −M
$U_4 = 0.2$	900 0	1300 0.5	1100 0.2	4.5	−3.6 6
$U_5 = -0.3$	−M M	1100 0	−3.3 3	1000 2.1	1200 1.9

Since all the shadow costs are either zero or -ive. So, this is the optimal table and the optimal sol: is given

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$900 \times 0.3 = 270$

$1400 \times 0.8 = 1120$

$1000 \times 2 = 2000$

$900 \times 0 = 0$

$1300 \times 0.5 = 650$

$1100 \times 0.2 = 220$

$1100 \times 0 = 0$

$1000 \times 2.1 = 2100$

$1200 \times 1.9 = 2280$

$= 8640$ Ans:-

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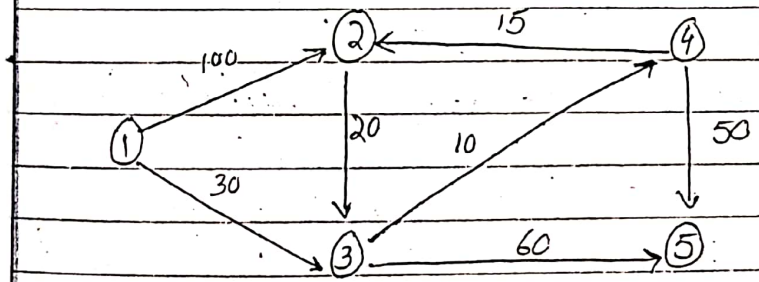
Dijkstra's Method:-

Q:- Determine the shortest distance b/w node 1 and every other node in the following network using dijkstra's method.



Sol:-

// Iteration 0:

Node 1 carries the permanent label as [0, -].

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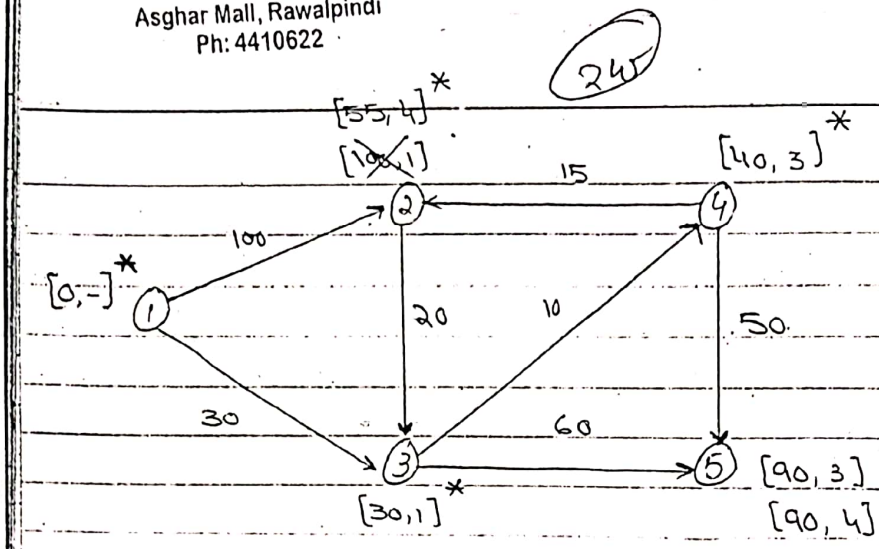

Iteration 1 :-

Node 2 and 3 can be reached from node 1. Label for these nodes are [100, 1] and [30, 1] respectively. Among these current labels, node 3 has the smallest distance.

d = min {30, 100}.

d = 30.

Thus node 3 is parmanently labeled

Iteration 2 :-

Node 4 and 5 can be reached from node 3. Label for these nodes are [40, 3] and [90, 3] respectively. Among these ~~nodes~~ current ~~nodes~~ labels, node 4 has the smallest distance.

d = min {40, 90}.

d = 40.

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(VVG)

Thus, node 4 is ~~paran~~ parmanently labeled.

Iteration 3:-

Node 2 and 5 can be reached from node 4. Label them nodes as [55, 4] and [90, 4] respectively. Among the current temporary labels, node 2 has the smallest distance.

$d = \min\{55, 90\}$

$d = 55$

Thus, node 2 is parmanently labeled.

The only remaining node is the node 5 which can't be reached from last parmanently node.

Therefore, node 5 can be reached either from node 3 or node 4.

So, the shortest distance of every node from node 1 is given below.

$. Distance blw Node 1 and 2 = 55 units.
and the path is.

$1 \rightarrow 3 \rightarrow 4 \rightarrow 2.$

Shortest distance blw node 1 and 3 = 30 units.
and the path is $1 \rightarrow 3$.

Shortest distance blw node 1 and 4 = 40 units.
and the path is

$1 \rightarrow 3 \rightarrow 4.$

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Shortest distance blw node 1 and 5 = 9 units and the path is:

1 → 3 → 5

OR 1 → 3 → 4 → 5.

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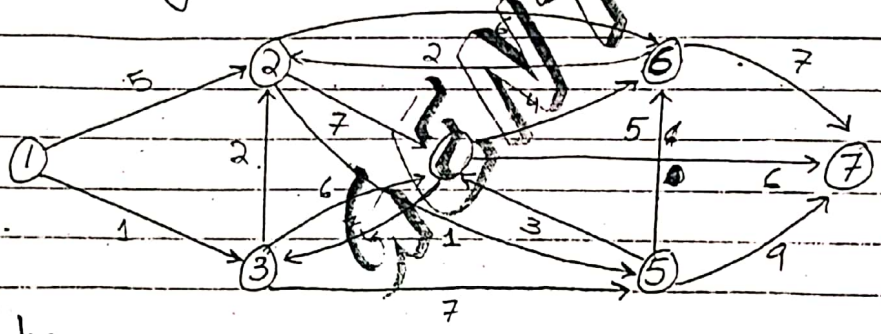
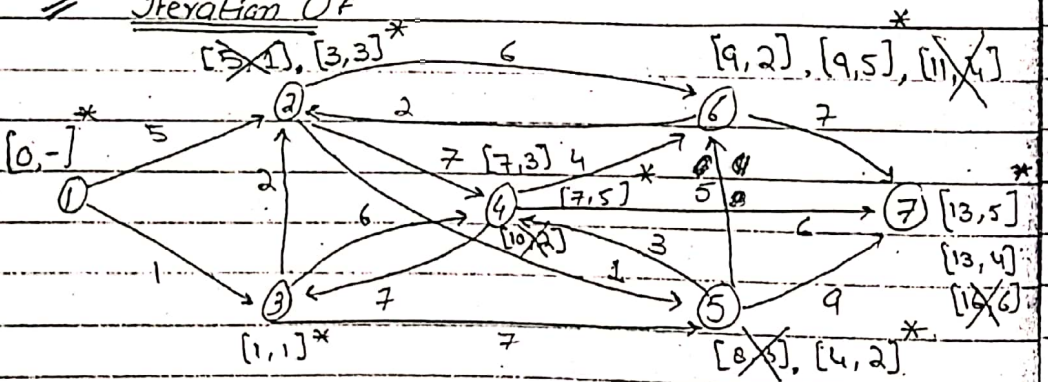
Q2.11 Determine the shortest distance blw node 1 and every other node in the following network.



Sol:-

Iteration 01



Label the node 1 as [0,-] as permanent label.

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Iteration 1:

Node 2 and 3 can be reached from node 1 directly. Label for these nodes are [5,1] and [1,1] respectively. Among these temporary label node 3 has smallest distance i-e.

$d = \min\{5, 1\}$.

$d = 1$

Thus, label node 3 as ~~para~~ permanent node.

Iteration 2:

Node 2, 4 and 5 can be reached directly from node 3. Label for these nodes are [3,3], [7,3] and [8,3] respectively. Among these ~~nodes~~ temporary labels node 2 has the smallest distance. i-e.

$d = \min\{3, 7, 8\}$

$d = 3$

Thus, label the node 2 as permanent label.

Iteration 3:

Node 4, 5 and 6 can be reached directly from node 2. Label for these nodes are [10,2], [4,2] and [9,2] respectively. Among these temporary labels node 5 has the smallest distance.

$d = \min\{10, 4, 9\}$.

$d = 4$

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Thus, mark the label node 5 as parmanent.

Iteration 4:

Node 4, 6 and 7 can be reached directly from node 5. Label for these nodes are [7, 5], [9, 5] and [13, 5] respectively. Among these labels node 4 has the smallest distance. i-e.

$d = \min\{7, 9, 13\}$

$d = 7$

Thus, mark the node 4 as parmanent.

Iteration 5:

Node 6 and 7 can be reach from node 4 directly. Label for these nodes are [11, 4] and [13, 4] respectively. Node 6 has the smallest distance. i-e.

$d = \min\{11, 13\}$

$d = 11$

Thus, mark the node 6 as parmanent. (Node 3 is already parmanently label. So, we can't move there.)

Iteration 6:

Node 7 is the only node, which can be reach from node 6. Label for this node is [16, 6]. So, at node 7 mark [13, 5] or [13, 4] as parmanent.

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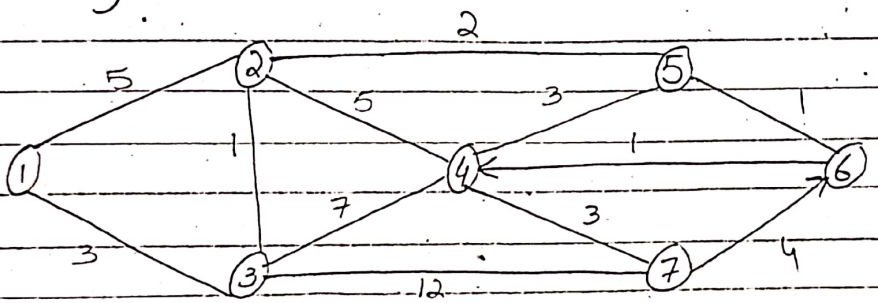
Hence, the shortest distance and the shortest route from node 1 to every other node are shown in the fig. above.

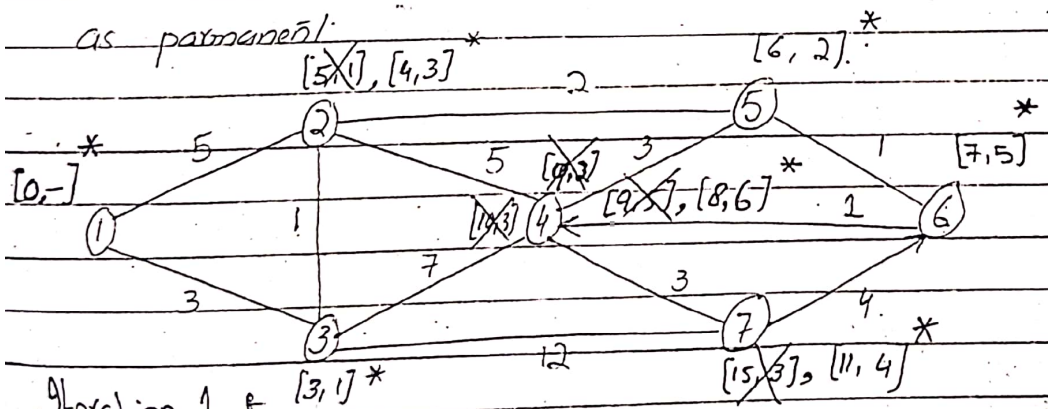
Q3.11 (Assignment):- Find the shortest distance between node 1 to every other node in the following network.



Sol:-

Iteration 0 :-

Label the node 1 as [0, -] as parmanent.



Iteration 1 :- Node 2 and 3 can be reach directly from node 1. Label for these nodes are [5, 1] and [3, 1] respectively. Among

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These nodes labels, node 3 has the smallest distance. i.e.

$d = \min\{5, 3\}$

$d = 3$, Thus, mark node 3 permanently.

Iteration 2:

Node 2, 4 and 7 can be reach directly from node 3. Label for these nodes are [4, 3], [10, 3] and [15, 3] respectively. Among these temporary labels, node 2 has the smallest distance. i.e.

$d = \min\{4, 10, 15\}$.

$\Rightarrow d = 4$.

Thus, mark the node 2 permanently.

Iteration 3:

Node 4 and 5 can be reach directly from node 2 Label for these nodes are [9, 2] and [6, 2] respectively. Among these temporary lables node 5 has the smallest distance i.e.

$d = \min\{9, 6\}$

$\Rightarrow d = 6$

Thus, make node 5 permanently

Iteration 4:

Node 4 and 6 can be reach from node 5 directly. Label for these node are [9, 5] and [7, 5] respectively. Among

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these temporary labels, node 6 has the smallest distance. i.e.

$d = \min\{9, 7\}$

$\Rightarrow d = 7$

Thus, mark node 6 permanently.

Iteration 5:

The only node which can be reach from node 6 is node 4. Label for this node is [8, 6]. Mark this node permanently.

Iteration 6:

The only node which can be reach from node 4 is node 7. Label for this node is 7. Marke this node permanently.

Hence, the shortest distance and the shortest route from node 1 to every other node is shown in the fig. above.

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-: SECTION III :-

-: Integer Linear Programming :-

Definition:-

Integer programming is the programming in which some or all the variables are restricted to be integers. They are commonly referred as mixed or pure integer programming.

Here, we discuss the following method of integer programming.

i) Fractional Cut Method
ii) Mixed Cut Method
iii) Branch And Bounded Method (B & B Method)

1. Fractional Cut Method:-

Formula for the Cut is.

S_i = Sum of all fraction − f_i

where;

f_i is maximum fraction in solution column.

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(Regular / Primal / Simplex Method).

Q1. Solve the following problem.

Max.

$$Z = 7x_1 + 9x_2$$

S/t

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

x_1, x_2 are non-negative integers.

By fractional Cut method.

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Sol: First we solve the problem by simplex method.

The given problem is ~~simp~~ standard form is.

Max.

$$Z - 7x_1 - 9x_2 = 0$$

S/t

$$-x_1 + 3x_2 + x_3 = 6$$

$$7x_1 + x_2 + x_4 = 35$$

In tabular Form:

Basic	x_1	x_2 ↓	x_3	x_4	Sol.	Ratio
Z	-7	-9	0	0	0	
← x_3	-1	$\boxed{3}$ P.E	1	0	6	$\frac{6}{3} = 2$
x_4	7	1	0	1	35	$\frac{35}{1} = 35$

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Basic	x_1	x_2	x_3	x_4	Sol	Ratio
z	-10	0	3	0	18	
x_2	$-1/3$	1	$1/3$	0	2	
← x_4	$\boxed{22/3}$ PE	0	$-1/3$	1	33	$\frac{33}{22/3} = \frac{9}{2}$.

Basic	x_1	x_2	x_3	x_4	Sol	Ratio
z	0	0	$28/11$	$15/11$	63	
x_2	0	1	$7/22$	$1/22$	$7/2 = 3(\frac{1}{2})$	
x_1	1	0	$-1/22$	$3/22$	$9/2 = 4(\frac{1}{2})$	

Since, all the coefficients of non-basic variables are either zero or +ive. So, this is the optimal table and the optimal solution is.

$z = 63$; $x_1 = 9/2$; $x_2 = 7/2$.

Since x_1 and x_2 are restricted to be integers. Therefore, we use the fractional cut method to make them integers

Since maximum of $f_i = \frac{1}{2}$.

So, those choose the any eq. for the fractional cut method

Consider,

x_2 - Equation.

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(All fractions must be +ve)

$$x_2 + \frac{7}{22}x_3 + \frac{1}{22}x_4 = \frac{7}{2}$$

$$x_2 + \left(0 + \frac{7}{22}\right)x_3 + \left(0 + \frac{1}{22}\right)x_4 = 3 + \frac{1}{2}$$

The corresponding fractional cut is.

$$S_1 = \frac{7}{22}x_3 + \frac{1}{22}x_4 - \frac{1}{2}$$

$$\Rightarrow S_1 - \frac{7}{22}x_3 - \frac{1}{22}x_4 = -\frac{1}{2}$$

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Add this cut to the previous optimal table and solve the problem by dual Simplex method

	B	x_1	x_2	x_3	x_4	S_1	Sol.
	Z	0	0	28/11	15/11	0	63
	x_2	0	1	7/22	1/22	0	7/2
	x_1	1	0	-1/22	3/22	0	9/2
←	S_1	0	0	(-7/22)	-1/22	1	-1/2
	Ratio			$\left\lvert\frac{28}{11} \div -\frac{22}{7}\right\rvert$ = |-8| = 8	$\left\lvert\frac{15}{11} \div -\frac{22}{1}\right\rvert$ = |-30| = 30		

B	x_1	x_2	x_3	x_4	S_1	Sol.
Z	0	0	0	1	8	59
x_2	0	1	0	0	1	3
x_1	1	0	0	1/7	-1/7	32/7 = $4\frac{4}{7}$
x_3	0	0	1	1/7	-22/7	11/7 = $1\frac{4}{7}$

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Now, the maximum of f_i is $\frac{4}{7}$.

i.e.

$$f_i = \frac{4}{7}$$

Consider Eq- x_1

$$x_1 + \frac{1}{7}x_4 - \frac{1}{7}S_1 = \frac{32}{7}$$

$$\Rightarrow x_1 + \left(0 + \frac{1}{7}\right)x_4 + \left(-1 + 1 - \frac{1}{7}\right)S_1 = \frac{32}{7}$$

$$\Rightarrow x_1 + \left(0 + \frac{1}{7}\right)x_4 + \left(-1 + \frac{6}{7}\right)S_1 = \frac{32}{7}\,4 + \frac{4}{7}$$

The corresponding fractional cut is

$$S_2 = \frac{1}{7}x_4 + \frac{6}{7}S_1 - \frac{4}{7}$$

$$\Rightarrow -\frac{1}{7}x_4 - \frac{6}{7}S_1 + S_2 = -\frac{4}{7}$$

Add this cut to the previous table and apply the dual simplex method.

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B	x_1	x_2	x_3	x_4	S_1	S_2	Sol.
Z	0	0	0	1	8	0	59
x_2	0	1	0	0	1	0	3
x_1	1	0	0	1/7	-1/7	0	32/7
x_3	0	0	1	1/7	-22/7	0	11/7
← S_2	0	0	0	(-1/7)	-6/7	1	-4/7
Ratio				|1/-1/7| = |-7| = 7	|8/-6/7| = 8×7/6 = 28/3		

B	x_1	x_2	x_3	x_4	S_1	S_2	Sol.
Z	0	0	0	0	2	7	55
x_2	0	1	0	0	1	0	3
x_1	1	0	0	0	-1	1	4
x_3	0	0	1	0	-4	1	1
x_4	0	0	0	1	6	-7	4

Since x_4 and x_2 are integers. So this is the optimal table and the optimal sol. is.

$Z = 55; \quad x_1 = 4, \quad x_2 = 3$

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Q2. (Assignment):- Solve the following problem by cutting method.

Max.

$$Z = 4x_1 + 6x_2 + 2x_3$$

S/t

$$4x_1 - 4x_2 \leq 5$$
$$-x_1 + 6x_2 \leq 5$$
$$-x_1 + x_2 + x_3 \leq 5$$

x_1 and x_2 are non-negative integers.

Sol:-

First, we solve the following problem by simplex method.

The given problem in standard form is.

Max.

$$Z - 4x_1 - 6x_2 - 2x_3 = 0$$

S/t

$$4x_1 - 4x_2 + x_4 = 5$$
$$-x_1 + 6x_2 + x_5 = 5$$
$$-x_1 + x_2 + x_3 + x_6 = 5$$

In tabular form is.

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B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio.
Z	-4	-6	-2	0	0	0	0	
x_4	4	-4	0	1	0	0	5	
← x_5	-1	(6) P.E	0	0	1	0	5	$\frac{5}{6}=0.83$
x_6	-1	1	1	0	0	1	5	$\frac{5}{1}=5$

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio.
Z	-5	0	-2	0	1	0	5	
← x_4	(10/3) P.E	0	0	1	2/3	0	25/3	5/2.
x_2	-1/6	1	0	0	1/6	0	5/6	
x_6	-5/6	0	1	0	-1/6	1	25/6	

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio.
Z	0	0	-2	3/2	2	0	35/2.	
x_1	1	0	0	3/10	1/5	0	5/2	
x_2	0	1	0	1/20	1/5	0	5/4	
← x_6	0	0	(1) P.E	1/4	0	1	25/4	$\frac{25}{4}/1=\frac{25}{4}$

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	0	0	2	2	2	30.	
x_1	1	0	0	3/10	1/5	0	$5/2=2\frac{1}{2}$	
x_2	0	1	0	1/20	1/5	0	$5/4=1\frac{1}{4}$	
x_3	0	0	1	1/4	0	1	$25/4=6\frac{1}{4}$	

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Since all the coefficients of non-basic variables are zero or +ive. So, this is the optimal table and optimal sol. is.

$$Z = 30; \quad x_1 = \frac{5}{2}; \quad x_2 = \frac{5}{4}, \quad x_3 = \frac{25}{4}$$

Since x_1, x_2 and x_3 are restricted to be integers. Therefore, we use the fractional cut method to make them integers.

Since maximum fraction is:

$$f_i = \frac{1}{2}$$

So, choose any one eq. for the fractional cut.

Consider;

x_1 – Eq.

$$x_1 + \frac{3}{10}x_4 + \frac{1}{5}x_5 = \frac{5}{2}.$$

$$\Rightarrow \left(x_1 + \left(0 + \frac{3}{10}\right)x_4 + \left(0 + \frac{1}{5}\right)x_5 = 2 + \frac{1}{2}\right).$$

Then the corresponding fractional cut is.

$$S_1 = \frac{3}{10}x_4 + \frac{1}{5}x_5 - \frac{1}{2}.$$

$$\Rightarrow \frac{3}{10}x_4 - \frac{1}{5}x_5 + S_1 = -\frac{1}{2}.$$

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Add this cut to the previous optimal table and solve the problem by dual simplex method.

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol.
z	0	0	0	2	2	2	0	30
x_1	1	0	0	3/10	1/5	0	0	5/2
x_2	0	1	0	1/20	1/5	0	0	5/4
x_3	0	0	1	1/4	0	1	0	25/4
← S_1	0	0	0	(-3/10) P.E	-1/5	0	1	-1/2
Ratio				|2/-3/10| = |-20/3|	|2/-1/5| = |-10|			
				= 20/3	= 10			

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol.	
z	0	0	0	0	2/3	2	20/3	80/3.	
x_1	1	0	0	0	0	0	1	2	
x_2	0	1	0	0	1/6	0	1/6	7/6 =	$1\frac{1}{6}$
x_3	0	0	1	0	-1/6	1	5/6	35/6 =	$5\frac{5}{6}$
x_4	0	0	0	1	2/3	0	-10/3	5/3 =	$1\frac{2}{3}$

Now;

max. of $f_i = \frac{5}{6}$

Consider,

Eq (3) as.

$$x_3 = \frac{1}{6}x_5 + x_6 + \frac{5}{6}S_1 = \frac{35}{6}$$

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$$x_3 + \left(1+1-\frac{1}{6}\right)x_5 + x_6 + \left(0+\frac{5}{6}\right)S_1 = 5+\frac{5}{6}$$

$$\Rightarrow x_3 + \left(-1+\frac{5}{6}\right)x_5 + x_6 + \left(0+\frac{5}{6}\right)S_1 = 5+\frac{5}{6}$$

Then the corresponding fractional cut is

$$S_2 = \frac{5}{6}x_5 + \frac{5}{6}S_1 - \frac{5}{6}$$

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$$\Rightarrow -\frac{5}{6}x_5 - \frac{5}{6}S_1 + S_2 = -\frac{5}{6}$$

Add this cut to the previous table and apply dual simplex method.

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	0	2/3	2	20/3	0	80/3
x_1	1	0	0	0	0	0	1	0	2
x_2	0	1	0	0	1/6	0	1/6	0	7/6
x_3	0	0	1	0	-1/6	-1	5/6	0	35/6
x_4	0	0	0	1	2/3	0	-10/3	0	5/3
← S_2	0	0	0	0	(-5/6) RE	0	-5/6	1	-5/6
Ratio					|2/3 / -5/6| = |4/5| = 4/5		|20/3 / -5/6| = |-8| = 8		

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B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	0	0	2	6	4/5	26
x_1	1	0	0	0	0	0	1	0	2
x_2	0	1	0	0	0	0	0	1/5	1
x_3	0	0	1	0	0	1	1	-1/5	6
x_4	0	0	0	1	0	0	-4	4/5	1
x_5	0	0	0	0	1	0	1	-6/5	1

Since x_1, x_2 and x_3 are all integers. So this is an optimal table and the optimal solution is.

$$Z = 26 \; ; \; x_1 = 2, \; x_2 = 1, \; x_3 = 6.$$

Q3. Solve the following problem by fractional cut method.

Max

$$Z = 3x_1 + x_2 + 3x_3.$$

S/t

$$-x_1 + 2x_2 + x_3 \leq 4$$

$$4x_2 - 3x_3 \leq 2.$$

$$x_1 - 3x_2 + 2x_3 \leq 3$$

x_1, x_2, x_3 are non-negative integers.

Compare the rounded optimal sol. and integer optimal solution.

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Sol:- First we solve the following problem by Simplex method.

The given problem in standard form is.

Max

$$Z - 3x_1 - x_2 - 3x_3 = 0$$

S/t

$$-x_1 + 2x_2 + x_3 + x_4 = 4$$
$$4x_2 - 3x_3 + x_5 = 2$$
$$x_1 - 3x_2 + 2x_3 + x_6 = 3.$$

In tabular form.

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	-3	-1	-3	0	0	0		
x_4	-1	2	1	1	0	0	4	
x_5	0	4	-3	0	1	0	2	
← x_6	(1)	-3	2	0	0	1	3	$\frac{3}{1} = 3$
	P.E							

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	0	-10	3	0	0	3	9	
x_4	0	-1	3	1	0	1	7	
← x_5	0	(4) P.E	-3	0	1	0	2	$\frac{2}{4} = \frac{1}{2}$
x_1	1	-3	2	0	0	1	3	

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B	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	0	0	-9/2	0	5/2	3	14	
x_4	0	0	(9/4) P.E	1	1/4	1	15/2	$\frac{15/2}{9/4} = 10/3$
x_2	0	1	-3/4	0	1/4	0	1/2	
x_1	1	0	-1/4	0	3/4	1	9/2	

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	0	0	2	3	5	29	
x_3	0	0	1	4/9	1/9	4/9	10/3. = $3(\frac{1}{3})$	
x_2	0	1	0	1/3	1/3	1/3	3	
x_1	1	0	0	1/9	7/9	10/9	16/3. = $5\frac{1}{3}$	

This is the optimal table and the rounded optimal sol. is:

$Z = 29$; $x_1 = 5$; $x_2 = 3$; $x_3 = 3$.

The sol. is not feasible because it does not satisfise the 2nd constraint. Since the variables are restricted to be integer. So we use the fractional cut method to make them integers.

Since maximum of f_i is $\frac{1}{3}$. So, we consider the

x_1 – Eq ~~constraint~~ arbitrarily.

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$$x_1 + \frac{1}{9}x_4 + \frac{7}{9}x_5 + \frac{10}{9}x_6 = \frac{16}{3}$$

$$\Rightarrow x_1 + \left(0+\frac{1}{9}\right)x_4 + \left(0+\frac{7}{9}\right)x_5 + \left(1+\frac{1}{9}\right)x_6 = 5+\frac{1}{3}$$

The corresponding fractional cut is:

$$S_1 = \frac{1}{9}x_4 + \frac{7}{9}x_5 + \frac{1}{9}x_6 - \frac{1}{3}$$

$$\Rightarrow S_1 - \frac{1}{9}x_4 - \frac{7}{9}x_5 - \frac{1}{9}x_6 = -\frac{1}{3}$$

Adding this cut to the previous table. So we apply dual Simplex method.

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol
Z	0	0	0	2	3	5	~~2~~ 0	29
x_3	0	0	1	4/9	1/9	4/9	0	$\frac{10}{3}=3(\frac{1}{3})$
x_2	0	1	0	1/3	1/3	1/3	0	3
x_1	1	0	0	1/9	7/9	10/9	0	$\frac{16}{3}=5(\frac{1}{3})$
← S_1	0	0	0	-1/9	(-7/9) PE	-1/9	1	-1/3
Ratio				|2/-1/9|	|3/-7/9|	|5/-1/9|		
				= 18	= 27/7	= 45		

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B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol.
Z	0	0	0	11/7	0	32/7	27/7	194/7
x_3	0	0	1	3/7	0	3/7	1/7	23/7 = 3(2/7)
x_2	0	1	0	2/7	0	2/7	3/7	20/7 = 2 6/7
x_1	1	0	0	0	0	1	1	5
x_5	0	0	0	1/7	1	1/7	-9/7	3/7 = 3/7

Since maximum of f_i is $\frac{6}{7}$

Considering x_2 - Eq.

$$x_2 + \frac{2}{7}x_4 + \frac{2}{7}x_6 + \frac{-3}{7}S_1 = 2 + \frac{6}{7}$$

Corresponding cut is

$$S_2 = \frac{2}{7}x_4 + \frac{2}{7}x_6 + \frac{3}{7}S_1 - \frac{6}{7}$$

$$\Rightarrow S_2 - \frac{2}{7}x_4 - \frac{2}{7}x_6 - \frac{3}{7}S_1 = -\frac{6}{7}$$

Add this cut to previous table Thus.

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	11/7	0	32/7	27/7	0	194/7
x_3	0	0	1	3/7	0	3/7	1/7	0	23/7
x_2	0	1	0	2/7	0	2/7	3/7	0	20/7
x_1	1	0	0	0	0	1	1	0	5
x_5	0	0	0	1/7	1	1/7	-9/7	0	3/7
← S_2	0	0	0	(-2/7) P.E	0	-2/7	-3/7	1	-6/7
Ratio				$\frac{11/7}{-2/7}$		$\frac{32/7}{-2/7}$	$\frac{27/7}{-3/7}$		
				= 11/2		= 16	= 9		

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B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	0	0	3	3/2	11/2	23
x_3	0	0	1	0	0	0	-1/2	3/2	2
x_2	0	1	0	0	0	0	0	1	2
x_1	1	0	0	0	0	1	1	0	5
x_5	0	0	0	0	1	0	-3/2	1/2	0
x_4	0	0	0	1	0	1	3/2	-7/2	3

Since x_1, x_2 and x_3 are integers. So, this is optimal table and optimal sol. is

$Z = 23$; $x_1 = 5$; $x_2 = 2$; $x_3 = 2$

Mixed Cut Method :-

Formula:- (جس پر کٹ لگے گا جو integers ہے)

$$S_i = \text{Sum of +ive fraction} + \left(\frac{f_i}{f_i - 1}\right)(\text{Sum of -ive fraction}) - f_i$$

(Cutting plane method is common name of Mixed cut and fractional cut method.)

Q:- 1. Solve the following problem by mixed cut method.

Max. $Z = 7x_1 + 9x_2$

s/t $-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

x_1, x_2 is non-negative integer ; $x_2 \neq 0$.

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Sol:- First we solve the problem by Simplex method.

The given problem in standard form is:

Max.

$$Z - 7x_1 - 9x_2 = 0$$

S/t

$$-x_1 + 3x_2 + x_3 = 6$$

$$7x_1 + x_2 + x_4 = 35$$

$$x_1, x_2, x_3, x_4 \geq 0$$

In Tabular form.

B	x_1	x_2	x_3	x_4	Sol	Ratio
Z	-7	-9	0	0	0	
← x_3	-1	(3) PE	1	0	6	$\frac{6}{3} = 2$
x_4	7	1	0	1	35	$\frac{35}{1} = 35$

B	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	-10	0	-3	0	18	
x_2	-1/3	1	1/3	0	2	
x_4	(22/3) PE	0	-1/3	1	33	$\frac{33}{22/3} = \frac{9}{2}$

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B	x_1	x_2	x_3	x_4	Sol.	Ratio
z	0	0	28/11	15/11	63.	
x_2	0	1	7/22	1/22	7/2. = $3\frac{1}{2}$	
x_1	1	0	-1/22	3/22	9/2. = $4\frac{1}{2}$	

This is the optimal table and optimal solution is:

$Z = 63$; $x_1 = 9/2$; $x_2 = 7/2$

Since x_4 is restricted to be integer. So, we use mixed cut method to make it integer.

Consider:

x_1 - Eq.

$$x_1 - \frac{1}{22}x_3 + \frac{3}{22}x_4 = \frac{9}{2}.$$

$$\Rightarrow x_1 - \frac{1}{22}x_3 + \frac{3}{22}x_4 = 4 + \frac{1}{2}.$$

The corresponding cut is.

$$S_1 = \frac{3}{22}x_4 + \frac{1/2}{1/2 - 1.}\left(-\frac{1}{22}x_3\right) - \frac{1}{2}.$$

$$= \frac{3}{22}x_4 + \frac{1/2}{+1/2}\left(+\frac{1}{22}x_3\right) - \frac{1}{2}$$

$$S_1 = \frac{3}{22}x_4 + \frac{1}{22}x_3 - \frac{1}{2}.$$

$$\Rightarrow S_1 - \frac{1}{22}x_3 - \frac{3}{22}x_4 = -\frac{1}{2}.$$

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Add this cut to the previous table and then apply the dual Simplex method:

	B	x_1	x_2	x_3	x_4	S_1	Sol
	Z	0	0	28/11	15/11	0	63
	x_2	0	1	7/22	1/22	0	7/2
	x_1	1	0	-1/22	3/22	0	9/2
←	S_1	0	0	-1/22	-3/22 (P.E)	1	-1/2
	Ratio			$\left\|\frac{28/11}{-1/22}\right\|$ = 56	$\left\|\frac{15/11}{-3/22}\right\|$ = 10		

	B	x_1	x_2	x_3	x_4	S_1	Sol
	Z	0	0	23/11	0	10	58
	x_2	0	1	10/33	0	1/3	10/3
	x_1	1	0	-1/11	0	1	4
	x_4	0	0	1/3	1	-22/3	11/3

This is the optimal table and optimal sol. is:

$Z = 58$; $x_1 = 4$; $x_2 = \frac{10}{3}$ Ans.

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Q2.4 Solve the following problem by mixed cut method.

Maximize.

$$Z = 3x_1 + x_2 + 3x_3$$

S/t

$$-x_1 + 2x_2 + x_3 \leq 4$$
$$x_1 - 3x_2 + 2x_3 \leq 3$$
$$4x_2 - 3x_3 \leq 2.$$

x_1, x_3 non-negative integer and $x_2 \geq 0$.

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Sol:-

First we solve the problem by ~~dual~~ regular simplex method.

The given problem in standard form is.

Max.

$$Z - 3x_1 - x_2 - 3x_3 = 0$$

S/t

$$-x_1 + 2x_2 + x_3 + x_4 = 4.$$
$$x_1 - 3x_2 + 2x_3 + x_5 = 3.$$
$$4x_2 - 3x_3 + x_6 = 2.$$

In Tabular Form is.

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio.
Z	-3	-1	-3	0	0	0	0	
x_4	-1	2	1	1	0	0	4	
← x_5	① P.E	-3	2	0	1	0	3	$\frac{3}{1} = 3$
x_6	0	4	-3	0	0	1	2	

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B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio.
Z	0	-10	3	0	3	0	9	
x_4	0	-1	3	1	1	0	7	
x_1	1	-3	2	0	1	0	3	
← x_6	0	(4) P.E	-3	0	0	1	2.	$\frac{2}{4}=\frac{1}{2}$

~~B x_1 x_2 x_3 x_4 x_5 x_6 Sol Ratio~~
~~Z~~
~~x_4 0 0 0 1 1 1 9~~
~~x_1~~
~~x_2 0 1 -9/4 0 0 1 2.~~

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	0	-9/2	0	3	5/2	14	
← x_4	0	0	(9/4) P.E	1	1	1/4	15/2	$\frac{15/2}{9/4}=\frac{10}{3}$
x_1	1	0	-1/4	0	1	3/4	9/2	
x_2	0	1	-3/4	0	0	1/4	1/2	

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	0	0	2	5	3	29	
x_3	0	0	1	4/9	4/9	1/9	$10/3=3\frac{1}{3}$	
x_1	1	0	0	1/9	10/9	7/9	$16/3=5\frac{1}{3}$	
x_2	0	1	0	1/3	1/3	1/3	3	

This is optimal table and optimal sol. is

$Z = 29$; $x_1 = 16/3$; $x_2 = 3$, $x_3 = 10/3$

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Since, x_1 and x_3 are restricted to be integers (non-negative).

Now, we use the mixed cut method to make it integers.

Here, maximum of $f_i \neq$

Consider,

x_1 - Equation.

$$x_1 + \frac{1}{9}x_4 + \frac{10}{9}x_5 + \frac{7}{9}x_6 = \frac{16}{3}$$

$$\Rightarrow x_1 + \left(0 + \frac{1}{9}\right)x_4 + \left(1 + \frac{1}{9}\right)x_5 + \left(0 + \frac{7}{9}\right)x_6 = 5 + \frac{1}{3}$$

The corresponding cut is

$$S_1 = \left(\frac{1}{9}x_4 + \frac{1}{9}x_5 + \frac{7}{9}x_6\right) + \frac{1/3}{1/3 - 1}(a) - \frac{1}{3}$$

$$S_1 = \frac{1}{9}x_4 + \frac{1}{9}x_5 + \frac{7}{9}x_6 - \frac{1}{3}$$

$$\Rightarrow S_1 - \frac{1}{9}x_4 - \frac{1}{9}x_5 - \frac{7}{9}x_6 = -\frac{1}{3}$$

Adding this cut to the previous table, then apply the dual simplex method.

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B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol.
Z	0	0	0	2	5	3	0	29
x_3	0	0	1	4/9	4/9	1/9	0	10/3
x_4	1	0	0	1/9	10/9	7/9	0	16/3
x_2	0	1	0	1/3	1/3	1/3	0	3
← S_1	0	0	0	-1/9	-1/9	-7/9 P.E	1	-1/3
Ratio.				|2/-1/9|	|5/-1/9|	|3/-7/9|	
				= 18	= 45	= 27/7		

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol.
Z	0	0	0	11/7	32/7	0	27/7	174/7
x_3	0	0	1	3/7	3/7	0	1/7	$23/7 = 3\frac{2}{7}$
x_1	1	0	0	0	1	0	1	5
x_2	0	1	0	2/7	2/7	0	3/7	$20/7 = 2\frac{6}{7}$
x_6	0	0	0	1/7	1/7	1	-9/7	3/7

Here, maximum of $f_i = \frac{6}{7}$

Consider:

x_2 - Eq.

$$x_2 + \frac{2}{7}x_4 + \frac{2}{7}x_5 + \frac{3}{7}S_1 = \frac{20}{7}$$

$$x_2 + \left(0 + \frac{2}{7}\right)x_4 + \left(0 + \frac{2}{7}\right)x_5 + \left(0 + \frac{3}{7}\right)S_1 = 2 + \frac{6}{7}$$

The ine corresponding cut is:

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$$S_2 = \frac{2}{7}x_4 + \frac{2}{7}x_5 + \frac{3}{7}S_1 + \frac{6/7}{1-6/7}(0) - \frac{6}{7}$$

$$\Rightarrow S_2 - \frac{2}{7}x_4 - \frac{2}{7}x_5 - \frac{3}{7}S_1 = -\frac{6}{7}$$

Adding this cut to previous table and apply dual Simplex method:

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	11/7	32/7	0	27/7	0	194/7
x_3	0	0	1	3/7	3/7	0	1/7	0	23/7
x_1	1	0	0	0	0	0	0	0	5
x_2	0	1	0	2/7	2/7	0	3/7	0	20/7
x_6	0	0	0	1/7	1/7	1	-9/7	0	3/7
← S_2	0	0	0	(-2/7) p.e	-2/7	0	-3/7	1	-6/7
Ratio				$\left\lvert\frac{11/7}{-2/7}\right\rvert$	$\left\lvert\frac{32/7}{-2/7}\right\rvert$		$\left\lvert\frac{27/7}{-3/7}\right\rvert$		
				=11/2	=16		=9		

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	0	3	0	3/2	11/2	23.
x_3	0	0	1	0	0	0	-1/2	3/2	2
x_1	1	0	0	0	1	0	1	0	5
x_2	0	1	0	0	0	0	0	1	2.
x_6	0	0	0	0	0	1	-3/2	1/2	0
x_4	0	0	0	1	1	0	3/2	-7/2	3

This is the optimal table and optimal sol is:

$Z = 23$; $x_1 = 5$, $x_2 = 2$, $x_3 = 2$ } Ans.

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Q3.11 Solve the following problem by mixed cut method.

Max.

$$Z = 4x_1 + 3x_2.$$

S/t

$$3x_1 + 4x_2 \leq 12.$$

$$x_1 + 2x_2 \leq 9$$

x_1, x_2 are non-ive integers.

Sol:-

First we solve the problem by Regular Simplex method.

The given problem in standard form is.

Max.

$$Z - 4x_1 - 3x_2 = 0.$$

S/t

$$3x_1 + 4x_2 + x_3 = 12.$$

$$x_1 + 2x_2 + x_4 = 9$$

$x_1, x_2, x_3, x_4 \geq 0.$

In Tabular Form.

B	x_1 ↓	x_2	x_3	x_4	Sol.	Ratio
Z	-4	-3	0	0	0	
← x_3	(3) P.E	4	1	0	12	$\frac{12}{3} = 4$
x_4	1	2	0	1	9	$\frac{9}{1} = 9$

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B.	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	7/3	4/3	0	16	
x_1	1	4/3	1/3	0	4	
x_4	0	2/3	-1/3	1	5	

This is the optimal table and optimal sol. is.

$Z = 16$; $x_1 = 4$; $x_2 = 0$.

Since x_1 and x_2 are non-negative integers. So this is required solution.

Q4:- Solve the following problem by mixed cut method.

Max $Z = x_1 - x_2$

s/t

$x_1 + 2x_2 \leq 4$

$6x_1 + 2x_2 \leq 9$

x_1, x_2 are non-ive integers.

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Sol:-

First we solve the following problem by regular simplex method in standard form.

Max

$Z - x_1 + x_2 = 0$

s/t

$x_1 + 2x_2 + x_3 = 4$

$6x_1 + 2x_2 + x_4 = 9$

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In Tabular form.

B	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	-1	1	0	0	0	
x_3	1	2	1	0	4	$\frac{4}{1} = 4$
← x_4	6 P.E	2	0	1	9	$\frac{9}{6} = \frac{3}{2}$

B	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	4/3	0	1/6	3/2	
x_3	0	5/3	1	-1/6	$5/2 = 2\frac{1}{2}$	
x_1	1	1/3	0	1/6	$3/2 = 1\frac{1}{2}$	

This is the optimal table and the optimal sol. is

$Z = \frac{3}{2}$; $x_1 = 3/2$; $x_2 = 0$.

Since, x_1 is not integer, so we ~~consider~~ make the x_1 integer by mixed cut method.

Then, the max. of $f_i = \frac{1}{2}$.

We consider x_1 - equation arbitrarly i-e.

$$x_1 + \frac{1}{3}x_2 + \frac{1}{6}x_4 = \frac{3}{2}.$$

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Then the corresponding cut is:

$$S_1 = \left(\frac{1}{3}x_2 + \frac{1}{6}x_4\right) + \left(\frac{\frac{1}{2}}{\frac{1}{2}-1}\right)(0) - \frac{1}{2}.$$

$$S_1 = \frac{1}{3}x_2 + \frac{1}{6}x_4 - \frac{1}{2}.$$

$$\Rightarrow S_1 - \frac{1}{3}x_2 - \frac{1}{6}x_4 = -\frac{1}{2}.$$

Add this cut to the previous table and solve by the dual Simplex method.

B	x_1	x_2	x_3	x_4	S_1	Sol.
Z	0	4/3	0	1/6	0	3/2
x_3	0	5/3	1	-1/6	0	5/2
x_1	1	1/3	0	1/6	0	3/2
← S_1	0	-1/3	0	(-1/6) P.E	1	-1/2
Ratio		$\left\|\frac{4/3}{-1/3}\right\| = 4$		$\left\|\frac{1/6}{-1/6}\right\| = 1$		

B	x_1	x_2	x_3	x_4	S_1	Sol.
Z	0	1	0	0	1	1
x_3	0	2	1	0	-1	3
x_1	1	0	0	0	1	1
x_4	0	2	0	1	-6	3

This is optimal table and optimal sol. is:

$Z = 1$; $x_1 = 1$; $x_2 = 0$

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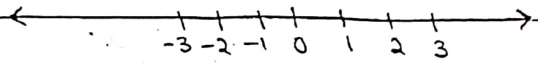
Brackel Function OR Greatest Integer Function :-

It is greatest integer not greater than x.



$[1.5] = 1$

Q:- Write a short note on Branch and Bound Method.

Branch And Bound Method :- (B & B Method) :-

In this method, the problem is 1st solved as a continuous model (By using regular simplex method). Suppose x_r is a variable which is restricted to integer value, whose optimum continuous value say x_r^* is fractional. Then, the range $[x_r^*] < x_r < [x_r^*] + 1$ can't includes any feasible integer sol. As a result a feasible integer.

$x_r \leq [x_r^*]$ or $x_r \geq [x_r^*] + 1$ ...(i)

value of x_r must satisfies the one of the two constraints we obtain two mutually exclusive problem by adding each of the above constraint to the original problem.

The original problem is said to be branched into two sub problems. Now each sub problem may be solved as a linear programme. If its optimal sol.

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is feasible w.r.t integer problem its solution is recorded as best one. So far available, otherwise the sub problem must be partitioned into two sub problems. When a better integer feasible solution is optimized for any sub problem it replaces to one at end.

The efficiency of the computation is increase by introducing the notation of bounding. According to this concept if the continuous optimum sol of a sub problem ~~get~~ yields a worse objective value, then the one associated with the best available integer sol., then there is no need to solve the problem any further.

Q1. Solve the following problem by B and B method.

Max.

$$Z = 2x_1 + 3x_2$$

S/t

$$5x_1 + 7x_2 \leq 35$$

$$4x_1 + 9x_2 \leq 36$$

x_1, x_2 are non-negative integers.

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Sol:- First, we solve the problem by the regular simplex method.

In standard Form.

Max

$$Z - 2x_1 - 3x_2 = 0.$$

S/t

$$5x_1 + 7x_2 + x_3 = 35$$
$$4x_1 + 9x_2 + x_4 = 36.$$

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In Tabular Form.

B	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	-2	-3	0	0	0	
x_3	5	7	1	0	35	$\frac{35}{7}=5$
← x_4	4	(9) P.E	0	1	36	$\frac{36}{9}=4$

B	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	-2/3	0	0	1/3	12	
← x_3	(17/9) P.E	0	1	-7/9	7	$\frac{63}{17}=3.7$
x_2	4/9	1	0	1/9	4	9

B	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	0	6/17	1/17	246/17	
x_1	1	0	9/17	-7/17	$\frac{63}{17}=3\frac{12}{17}$	
x_2	0	1	-4/17	5/17	$\frac{40}{17}=2\frac{6}{17}$	

This is the optimal table.

Since x_1 and x_2 are restricted to be

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integers, so we use B and B method to make them integers.

Now, we select the x_2 Equation arbitrarly for branching.

Then, the two problems are created by the restriction.

As;

$$x_2 \leq \left[\frac{40}{17}\right] = [2.3] \quad \text{or;} \quad x_2 \geq \left[\frac{40}{17}\right] + 1$$

$$\Rightarrow x_2 \leq 2 \quad \text{or;} \quad x_2 \geq 3.$$

Sub Problem 1 (Lower bound).

Max.

$$Z = 2x_1 + 3x_2.$$

s/t

$$5x_1 + 7x_2 \leq 35$$

$$4x_1 + 9x_2 \leq 36.$$

$$x_2 \leq 2.$$

Sub Problem 2 (Upper bound).

Max.

$$Z = 2x_1 + 3x_2.$$

s/t

$$5x_1 + 7x_2 \leq 35$$

$$4x_1 + 9x_2 \leq 36$$

$$x_2 \geq 3$$

Now, we solve the sub problem 1 :-

Consider;

$$x_2 \leq 2.$$

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$\Rightarrow \quad x_2 + S_1 = 2.$

$\Rightarrow \quad x_2 = 2 - S_1.$

From the last optimal table.

$$x_2 - \frac{4}{17}x_3 + \frac{5}{17}x_4 = \frac{40}{17}$$

$$\Rightarrow 2 - S_1 - \frac{4}{17}x_3 + \frac{5}{17}x_4 = \frac{40}{17}$$

$$\Rightarrow S_1 + \frac{4}{17}x_3 - \frac{5}{17}x_4 = 2 - \frac{40}{17}.$$

$$\Rightarrow S_1 + \frac{4}{17}x_3 - \frac{5}{17}x_4 = -\frac{6}{17}$$

Adding this cut to the previous optimal table and apply dual simplex method.

↓

B	x_1	x_2	x_3	x_4	S_1	Sol.
Z	0	0	6/17	1/17	0	246/17
x_1	1	0	9/17	-7/17	0	63/17
x_2	0	1	-4/17	5/17	0	40/17
← S_1	0	0	4/17	(-5/17) p.t	1	-6/17.
Ratio				1/17 / -5/17		

$= 1/5$

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B	x_1	x_2	x_3	x_4	S_1	Sol.
z	0	0	2/5	0	1/5	72/5
x_1	1	0	1/5	0	-7/5	21/5
x_2	0	1	0	0	1	2
x_4	0	0	-4/5	1	-17/5	6/5

Since, x_1 has fractional value, it is now selected for branching and two sub problems created by introducing the constraints.

$$x_1 \leq \left[\frac{21}{5}\right] = [4.2], \text{ or } \quad x_1 \geq \left[\frac{21}{5}\right] + 1$$

$$\Rightarrow x_1 \leq 4 \quad \text{or} \quad x_1 \geq 5$$

Then, the two new sub problems are

Sub Problem 1 & 2

Max.

$$Z = 2x_1 + 3x_2$$

s/t

$$5x_1 + 7x_2 \leq 35$$
$$4x_1 + 9x_2 \leq 36$$
$$x_2 \leq 2$$
$$x_1 \leq 4$$

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Sub Problem 1b

Max.

$$Z = 2x_1 + 3x_2$$

s.t

$$5x_1 + 7x_2 \leq 35$$
$$4x_1 + 9x_2 \leq 36$$
$$x_2 \leq 2$$
$$x_1 \geq 5$$

Now, we solve the problem 1a.

Consider: ~~x_1~~ Eq

$$x_1 \leq 4$$
$$x_1 + S_2 = 4$$
$$\Rightarrow x_1 = 4 - S_2$$

From the last table of problem 1.

Consider x_1 - Eq.

$$x_1 + \frac{1}{5}x_3 - \frac{7}{5}S_1 = \frac{21}{5}$$

$$\Rightarrow 4 - S_2 + \frac{1}{5}x_3 - \frac{7}{5}S_1 = \frac{21}{5}$$

$$\Rightarrow S_2 - \frac{1}{5}x_3 + \frac{7}{5}S_1 = 4 - \frac{21}{5}$$

$$\Rightarrow S_2 - \frac{1}{5}x_3 + \frac{7}{5}S_1 = -\frac{1}{5}$$

Adding this cut to the previous table of sub problem 1. and apply dual Simplex method.

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B	x_1	x_2	x_3	x_4	S_1	S_2	Sol
Z	0	0	2/5	0	1/5	0	72/5
x_1	1	0	1/5	0	-7/5	0	21/5
x_2	0	1	0	0	1	0	2
x_4	0	0	-4/5	1	-17/5	0	6/5
← S_2	0	0	(-1/5) P.E	0	7/5	1	-1/5
Ratio			$\left\lvert \frac{2/5}{-1/5} \right\rvert$				
			$= 2$				

B	x_1	x_2	x_3	x_4	S_1	S_2	Sol
Z	0	0	0	0	3	2	14
x_1	1	0	0	0	0	1	4
x_2	0	1	0	0	1	0	2
x_4	0	0	0	1	-9	-4	2
x_3	0	0	1	0	-7	-5	1

Since x_1 and x_2 are integers.
Therefore, integer sol so far obtained is.

$Z = 14$; $x_1 = 4$; $x_2 = 2$.

Now, we solve the sub problem 1b.
Consider.

$x_1 \geq 5$

$\Rightarrow -x_1 \leq -5$

$\Rightarrow -x_1 + S_3 = -5$

$\Rightarrow x_1 = 5 + S_3$

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Consider x_1- Eq. from optimal table of sub problem 1.

$$x_1 + \frac{1}{5}x_3 - \frac{7}{5}S_1 = \frac{21}{5}$$

$$\Rightarrow 5 + S_3 + \frac{1}{5}x_3 - \frac{7}{5}S_1 = \frac{21}{5}$$

$$\Rightarrow S_3 + \frac{1}{5}x_3 - \frac{7}{5}S_1 = \frac{21}{5} - 5$$

$$\Rightarrow S_3 + \frac{1}{5}x_3 - \frac{7}{5}S_1 = -\frac{4}{5}$$

Adding this cut to the optimal table of sub problem 1.

B	x_1	x_2	x_3	x_4	S_1	S_3	Sol.
Z	0	0	2/5	0	1/5	0	72/5
x_1	1	0	1/5	0	-7/5	0	21/5
x_2	0	1	0	0	1	0	2
x_4	0	0	-4/5	1	-17/5	0	6/5
$\leftarrow S_3$	0	0	1/5	~~2/5~~	(-7/5) P.E	1	-4/5
Ratio					$\left\lvert\frac{1/5}{-7/5}\right\rvert$		
					$= 1/7$		

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B	x_1	x_2	x_3	x_4	S_1	S_3	Sol.
Z	0	0	3/7	0	0	1/7	100/7
x_4	1	0	0	0	0	-1	5
x_2	0	1	1/7	0	0	5/7	10/7
x_4	0	0	-9/7	1	0	-17/5	22/7
S_1	0	0	-1/7	0	1	-5/7	4/7

As the sol. yield by this sub problem is fractionally higher than lower bound any branch from this sub problem can't yield a better value than the lower bound across the all coefficient of the objective functions are integers.

Sub Problem 2

Max.

$$Z = 2x_1 + 3x_2$$

S/t

$$5x_1 + 7x_2 \leq 35$$

$$4x_1 + 9x_2 \leq 36$$

$$x_2 \geq 3$$

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Consider:

$$x_2 \geq 3$$

$$-x_2 \leq -3$$

$$-x_2 + S_4 = -3$$

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$\Rightarrow x_2 = 3 + S_4$

Consider;

x_2 - Eq. from optimal table of the given problem.

$$x_2 - \frac{4}{17}x_3 + \frac{5}{17}x_4 = \frac{40}{17}$$

$$\Rightarrow 3 + S_4 - \frac{4}{17}x_3 + \frac{5}{17}x_4 = \frac{40}{17}$$

$$\Rightarrow S_4 - \frac{4}{17}x_3 + \frac{5}{17}x_4 = \frac{40}{17} - 3$$

$$\Rightarrow S_4 - \frac{4}{17}x_3 + \frac{5}{17}x_4 = -\frac{11}{17}$$

Adding this cut in the optimal table of given problem:

B	x_1	x_2	x_3	x_4	S_4	Sol.
Z	0	0	6/17	1/17	0	246/17
x_1	1	0	9/17	-7/17	0	63/17
x_2	0	1	-4/17	5/17	0	40/17
$\leftarrow S_4$	0	0	(-4/17) P.E	5/17	1	-11/17
Ratio			$\left\lvert\frac{6/17}{-4/17}\right\rvert$			
			$= \frac{3}{2}$			

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B	x_1	x_2	x_3	x_4	S_4	Sol.
Z	0	0	0	1/2	3/2	27/2
x_1	1	0	0	1/4	9/4	9/4
x_2	0	1	0	0	-1	3
x_3	0	0	1	-5/4	-17/4	11/4

This sol (Z) less than the lower bound. So. there is no need to investigate it further.

Therefore, optimal sol. is given by. sub problem 1a. i-e.

$Z = 14 \; ; \; x_1 = 4 \; ; \; x_2 = 2.$

Q2: Solve the following problem by B and B method.

Max.

$$Z = 3x_1 + x_2 + 3x_3$$

S/t

$$-x_1 + 2x_2 + x_3 \leq 4$$
$$4x_2 - 3x_3 \leq 2.$$
$$x_1 - 3x_2 + 2x_3 \leq 3$$

x_1 and x_3 are non-ive integers and $x_2 \geq 0$.

Sol: First we apply the regular Simplex method.

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In standard Form.

$$Z - 3x_1 - x_2 - 3x_3 = 0$$

S/t

$$-x_1 + 2x_2 + x_3 + x_4 = 4$$
$$4x_2 - 3x_3 + x_5 = 2$$
$$x_1 - 3x_2 + 2x_3 + x_6 = 3$$

In Tabular form.

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio:
Z	-3	-1	-3	0	0	0	0	
x_4	-1	2	1	1	0	0	4	
x_5	0	4	-3	0	1	0	2	
← x_6	(1) P.E	-3	2	0	0	1	3	$\frac{3}{1} = 3$

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio:
Z	0	-10	3	0	0	3	9	
x_4	0	-1	3	1	0	1	7	
← x_5	0	(4) P.E	-3	0	1	0	2	$\frac{2}{4} = \frac{1}{2}$
x_1	1	-3	2	0	0	1	3	

B	x_1	x_2	x_3	x_4	x_5	x_6	Sol.	Ratio
Z	0	0	-9/2	0	5/2	3	14	
← x_4	0	0	(9/4) P.E	1	1/4	1	15/2	$\frac{15/2}{9/4} = \frac{10}{3}$
x_2	0	1	-3/4	0	1/4	0	1/2	
x_1	1	0	-1/4	0	3/4	1	9/2	

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B	x_1	x_2	x_3	x_4	x_5	x_6	Sol	Ratio
Z	0	0	0	2	3	5	29	
x_3	0	0	1	4/9	1/9	4/9	$10/3 = 3\frac{1}{3}$	
x_2	0	1	0	1/3	1/3	1/3	3	
x_1	1	0	0	1/9	7/9	10/9	$16/3 = 5\frac{1}{3}$	

Since x_1 and x_3 are restricted to be non-ive integers. So, we use B and B method to make them integers. So, we consider x_1 = Eq arbitrarily. Then the two problems are to be created as

$x_1 \leq \left[\frac{16}{3}\right]$ and $x_1 \geq \left[\frac{16}{3}\right] + 1$

$\Rightarrow x_1 \leq 5$ and $x_1 \geq 6$

Sub Problem 1: (Lower Bound).

Max

$$Z = 3x_1 + x_2 + 3x_3$$

S.t

$$-x_1 + 2x_2 + x_3 \leq 4$$

$$4x_2 - 3x_3 \leq 2.$$

$$3x_2 + 2x_3 \leq 3$$

$$x_1 \leq 5$$

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Sub Problem 2b (Upper bound)

Max

$$Z = 3x_1 + x_2 + 3x_3$$

S.t

$$-x_1 + 2x_2 + x_3 \leq 4$$
$$4x_2 - 3x_3 \leq 2$$
$$x_1 - 3x_2 + 2x_3 \leq 3$$
$$x_1 \geq 6.$$

First we solve the sub problem 1.

Consider:

$$x_1 \leq 5$$
$$\Rightarrow x_1 + S_1 = 5$$
$$\Rightarrow x_1 = 5 - S_1$$

From the last optimal table. Consider.

$$x_1 + \frac{1}{9}x_4 + \frac{7}{9}x_5 + \frac{10}{9}x_6 = \frac{16}{3}$$

$$\Rightarrow 5 - S_1 + \frac{1}{9}x_4 + \frac{7}{9}x_5 + \frac{10}{9}x_6 = \frac{16}{3}$$

$$\Rightarrow S_1 - \frac{1}{9}x_4 - \frac{7}{9}x_5 - \frac{10}{9}x_6 = 5 - \frac{16}{3}$$

$$\Rightarrow S_1 - \frac{1}{9}x_4 - \frac{7}{9}x_5 - \frac{10}{9}x_6 = -\frac{1}{3}$$

Adding this cut to the previous optimal table. Then

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$$x_3 + \frac{3}{7}x_4 + \frac{2}{7}x_6 + \frac{1}{7}S_1 = \frac{23}{7}$$

$$\Rightarrow 3 - S_2 + \frac{3}{7}x_4 + \frac{2}{7}x_6 + \frac{1}{7}S_1 = \frac{23}{7}$$

$$\Rightarrow S_2 - \frac{3}{7}x_4 - \frac{2}{7}x_6 - \frac{1}{7}S_1 = 3 - \frac{23}{7}$$

$$\Rightarrow S_2 - \frac{3}{7}x_4 - \frac{2}{7}x_6 - \frac{1}{7}S_1 = -\frac{2}{7}$$

Adding this col to the previous table of sub problem 1.

B	x_1	x_2	x_3	x_4	x_5	x_6 ↓	S_1	S_2	Sol.
Z	0	0	0	11/7	0	5/7	27/7	0	194/7
x_3	0	0	1	3/7	0	2/7	1/7	0	23/7
x_2	0	1	0	2/7	0	-1/7	3/7	0	20/7
x_1	1	0	0	0	0	0	1	0	5
x_5	0	0	0	1/7	1	10/7	-9/7	0	3/7
← S_2	0	0	0	-3/7	0	(-2/7) P.E	-1/7	1	-2/7
Ratio-				$\left\lvert\frac{11/7}{-3/7}\right\rvert$		$\left\lvert\frac{5/7}{-2/7}\right\rvert$	$\left\lvert\frac{27/7}{-1/7}\right\rvert$		
				$=\frac{11}{3}$		$=\frac{5}{2}$	$=27$		

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	1/2	0	0	7/2	5/2	27
x_3	0	0	1	0	0	0	0	1	3
x_2	0	1	0	1/2	0	0	1/2	-1/2	3
x_1	1	0	0	0	0	0	1	0	5
x_5	0	0	0	-2	1	0	-2	2	-1
x_6	0	0	0	3/2	0	1	1/2	-7/2	1

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Since x_1 and x_2 are restricted to be integers (non-ive). So, we use the B and B method to make them integers. Then, the two problems are created by the restriction.

As consider:

$x_3 \leq \left[\frac{10}{3}\right]$; $x_3 \geq \left[\frac{10}{3}\right] + 1$

$x_3 \leq 3$; $x_3 \geq 4$.

Sub Problem 1: (Lower Bound).

Max.

$$Z = 3x_1 + x_2 + 3x_3.$$

s/t

$$-x_1 + 2x_2 + x_3 \leq 4$$
$$4x_2 - 3x_3 \leq 2.$$
$$x_1 - 3x_2 + 2x_3 \leq 3$$
$$x_3 \leq 3$$

Sub Problem 2: (Upper Bound).

Max.

$$Z = 3x_1 + x_2 + 3x_3$$

s/t

$$-x_1 + 2x_2 + x_3 \leq 4$$
$$4x_2 - 3x_3 \leq 2$$
$$x_1 - 3x_2 + 2x_3 \leq 3$$
$$x_3 \geq 4$$

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First we solve the sub problem 1.

Consider

$x_3 \leq 3.$

$x_3 + S_1 = 3.$

$\Rightarrow x_3 = 3 - S_1.$

Consider the x_3 - Eq from optimal table of given problem.

$$x_3 + \frac{4}{9}x_4 + \frac{1}{9}x_5 + \frac{4}{9}x_6 = \frac{10}{3}$$

$$\Rightarrow 3 - S_1 + \frac{4}{9}x_4 + \frac{1}{9}x_5 + \frac{4}{9}x_6 = \frac{10}{3}.$$

$$\Rightarrow S_1 - \frac{4}{9}x_4 - \frac{1}{9}x_5 - \frac{4}{9}x_6 = 3 - \frac{10}{3} = -\frac{1}{3}$$

$$\Rightarrow S_1 - \frac{4}{9}x_4 - \frac{1}{9}x_5 - \frac{4}{9}x_6 = -\frac{1}{3}.$$

Adding this to optimal table of given problem.

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol.
Z	0	0	0	2	3	5	0	29
x_3	0	0	1	4/9	1/9	4/9	0	10/3
x_2	0	1	0	1/3	1/3	1/3	0	3
x_1	1	0	0	1/9	7/9	10/9	0	16/3
← S_1	0	0	0	(-4/9) P.E	-1/9	-4/9	1	-1/3
Ratio				$\left\lvert 2/\frac{-4}{9} \right\rvert$	$\left\lvert \frac{3}{-1/9} \right\rvert$	$\left\lvert \frac{5}{-4/9} \right\rvert$		
				$= \frac{9}{2}$	$= 27$	$= \frac{45}{4}$		

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B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	Sol.
Z	0	0	0	0	5/2	3	9/2	55/2
x_3	0	0	1	0	0	0	1	3
x_2	0	1	0	0	1/4	0	3/4	11/4
x_1	1	0	0	0	3/4	1	1/4	21/4
x_4	0	0	0	1	1/4	1	-9/4	3/4

Since x_1 is in fraction. So, we again consider:

$x_1 \leq \left[\frac{21}{4}\right]$ and $x_1 \geq \left[\frac{21}{4}\right] + 1$

$\Rightarrow x_1 \leq 5$ and $x_1 \geq 6$.

Then, the two new sub problems are.

Sub Problem 1a:

Max.

$$Z = 3x_1 + x_2 + 3x_3$$

s.t

$$-x_1 + 2x_2 + x_3 \leq 4$$
$$4x_2 - 3x_3 \leq 2$$
$$x_1 - 3x_2 + 2x_3 \leq 3$$
$$x_3 \leq 3$$
$$x_1 \leq 5$$

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Sub Problem 1b:

Max. $Z = 3x_1 + x_2 + 3x_3$

s.t $-x_1 + 2x_2 + x_3 \leq 4$

$4x_2 - 3x_3 \leq 2$

$x_1 - 3x_2 + 2x_3 \leq 3$

$x_3 \leq 3$; ~~$x_1 \leq 5$~~ $x_1 \geq 6$

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Now, we solve the sub problem 1a.

Consider;

$$x_1 \leq 5$$

$$\Rightarrow x_1 + S_2 = 5$$

$$\Rightarrow x_1 = 5 - S_2.$$

Consider x_1 Eq from optimal table of Sub P.1.

$$x_1 + \frac{3}{4}x_5 + x_6 + \frac{1}{4}S_1 = \frac{21}{4}$$

$$\Rightarrow 5 - S_2 + \frac{3}{4}x_5 + x_6 + \frac{1}{4}S_1 = \frac{21}{4}$$

$$\Rightarrow S_2 - \frac{3}{4}x_5 - x_6 - \frac{1}{4}S_1 = -\frac{1}{4}$$

Add this cut to the optimal table of Sub P.1.

B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	0	5/2	3	9/2	0	55/2
x_3	0	0	1	0	0	0	1	0	3
x_2	0	1	0	0	1/4	0	3/4	0	11/4
x_1	1	0	0	0	3/4	1	1/4	0	21/4
x_4	0	0	0	1	1/4	1	-9/4	0	3/4
$\leftarrow S_2$	0	0	0	0	-3/4	(-1)	-1/4	1	-1/4
					$\left\lvert\frac{5/2}{-3/4}\right\rvert$	$\left\lvert\frac{3}{-1}\right\rvert$	$\left\lvert\frac{9/2}{-1/4}\right\rvert$		
					$= \frac{10}{3}$	$= 3$	18		

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B	x_4	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	0	1/4	0	15/4	0	107/4
x_3	0	0	1	0	0	0	1	0	3
x_2	0	1	0	0	1/4	0	3/4	0	11/4
x_1	1	0	0	0	0	0	0	1	5
x_4	0	0	0	1	-1/2	0	-5/2	1	1/2
x_6	0	0	0	0	3/4	1	1/4	-1	1/4

Since x_1 and x_3 are integers. So, this sol. is acceptable.

$\therefore Z = \frac{107}{4}$; $x_1 = 5$, $x_3 = 3$, $x_2 = \frac{11}{4}$

Now, we solve sub problem 1b.

Consider:

$$x_1 \not\geq 6$$

$$\Rightarrow -x_1 \leq -6$$

$$\Rightarrow -x_1 + S_3 = -6$$

$$\Rightarrow x_1 = 6 + S_3$$

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Consider x_1 - Eq from optimal table of sub-P1.

$$x_1 + \frac{3}{4}x_5 + x_6 + \frac{1}{4}S_1 = \frac{21}{4}$$

$$6 + S_3 + \frac{3}{4}x_5 + x_6 + \frac{1}{4}S_1 = \frac{21}{4}$$

$$\Rightarrow S_3 + \frac{3}{4}x_5 + x_6 + \frac{1}{4}S_1 = -\frac{3}{4}$$

Adding this to the optimal table of the sub problem 1

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B	x_1	x_2	x_3	x_4	x_5	x_6	S_1	S_2	Sol.
Z	0	0	0	0	5/2	3	9/2	0	55/2
x_3	0	0	1	0	0	0	1	0	3
x_2	0	1	0	0	1/4	0	3/4	0	11/4
x_1	1	0	0	0	3/4	1	1/4	0	21/4
x_4	0	0	0	1	1/4	1	-9/4	0	3/4
$\leftarrow S_2$	0	0	0	0	3/4	1	1/4	1	-3/4

This problem has no feasible sol. because it has no entering variable.

Now, we solve the sub problem 2.

Consider:

$$x_3 \geq 4$$

$$-x_3 \leq -4$$

$$-x_3 + S_4 = -4$$

$$\Rightarrow x_3 = 4 + S_4$$

Consider, x_3 - Eq from optimal table of given problem.

$$x_3 + \frac{4}{9}x_4 + \frac{1}{9}x_5 + \frac{4}{9}x_6 = \frac{10}{3}$$

$$4 + S_4 + \frac{4}{9}x_4 + \frac{1}{9}x_5 + \frac{4}{9}x_6 = \frac{10}{3}$$

$$\Rightarrow S_4 + \frac{4}{9}x_4 + \frac{1}{9}x_5 + \frac{4}{9}x_6 = -\frac{2}{3}$$

Adding this to the optimal table of the given problem.

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B	x_1	x_2	x_3	x_4	x_5	x_6	S_4	Sol.
z	0	0	0	2	3	5	0	29
x_3	0	0	1	4/9	1/9	4/9	0	10/3
x_2	0	1	0	1/3	1/3	1/3	0	3
x_1	1	0	0	1/9	7/9	10/9	0	16/3
$\leftarrow S_4$	0	0	0	4/9	1/9	4/9	1	-2/3

This problem has no feasible solution because it has no entering variable.

Hence the optimal sol. is.

$$Z = \frac{107}{4}; \quad x_1 = 5, x_3 = 3; \quad x_2 = \frac{11}{4}$$

Q3.11 Solve the following problem by B and B method.

Max.

$$Z = x_1 + x_2$$

s/t

$$2x_1 + 5x_2 \leq 16$$

$$6x_1 + 5x_2 \leq 30$$

x_1, x_2 are non-ive integers.

Sol:-

First we solve the problem by regular simplex method.

In Standard Form.

$$Z - x_1 - x_2 = 0$$

s/t

$$2x_1 + 5x_2 + x_3 = 16$$

$$6x_1 + 5x_2 + x_4 = 30$$

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In Tabular form.

B	x_1	x_2	x_3	x_4	Sol.	Ratio.
Z	-1	-1	0	0	0	
x_3	2	5	1	0	16	$\frac{16}{2}=8$
← x_4	(6) P.E	5	0	1	30	$\frac{30}{6}=5$

B	x_1	x_2	x_3	x_4	Sol.	Ratio.
Z	0	-1/6	0	1/6	5	
← x_3	0	(10/3) P.E	1	-1/3	6	9/5
x_1	1	5/6	0	1/6	5	$\frac{5}{1}=5$

B	x_1	x_2	x_3	x_4	Sol.	Ratio
Z	0	0	1/20	3/20	53/10	
x_2	0	1	3/10	-1/10	9/5	
x_1	1	0	-1/4	1/4	7/2	

Since x_1 and x_2 are restricted to be integers. So, we apply B and B method.

Consider:

$x_2 \leq \left[\frac{9}{5}\right]$ and $x_2 \geq \left[\frac{9}{5}\right]+1$

$x_2 \leq 1$ and $x_2 \geq 2$

Sub Problem 1 (Lower Bound).

Max.

$$Z = x_1 + x_2$$

s/t

$$2x_1 + 5x_2 \leq 16$$

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$$6x_1 + 5x_2 \leq 30$$
$$x_2 \leq 1$$

Sub Problem 2 (Upper Bound).

Max
$$Z = x_1 + x_2.$$

S/t
$$2x_1 + 5x_2 \leq 16$$
$$6x_1 + 5x_2 \leq 30$$
$$x_2 \geq 1$$

First we solve sub problem 1.

Consider:
$$x_2 \leq 1$$
$$x_2 + S_1 = 1$$
$$\Rightarrow x_2 = 1 - S_1$$

Consider x_2 - Eq from optimal table of given problem.

$$x_2 + \frac{3}{10}x_3 - \frac{1}{10}x_4 = \frac{9}{5}$$

$$1 - S_1 + \frac{3}{10}x_3 - \frac{1}{10}x_4 = \frac{9}{5}$$

$$\Rightarrow S_1 - \frac{3}{10}x_3 + \frac{1}{10}x_4 = -\frac{4}{5}.$$

Adding this cut to the optimal table of the given problem.

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B	x_1	x_2	x_3	x_4	S_1	Sol.
Z	0	0	1/20	3/20	0	53/10
x_2	0	1	3/10	-1/10	0	9/5
x_1	1	0	-1/4	1/4	0	7/2
← S_1	0	0	(-3/10) P.E	1/10	1	-4/5
Ratio			$\left\lvert \frac{1/20}{-3/10} \right\rvert$			
			= 1/6.			

B	x_1	x_2	x_3	x_4	S_1	Sol.
Z	0	0	0	1/6	1/6	31/6
x_2	0	1	0	0	1	1
x_1	1	0	0	1/6	-5/6	25/6
x_3	0	0	1	-1/3	-10/3	8/3.

Since, x_1 is again in fraction. So we again apply B and B method.

Consider

$x_1 \leq \left[\frac{25}{6}\right]$ and $x_1 \geq \left[\frac{25}{6}\right] + 1$

$x_1 \leq 4$ and $x_1 \geq 5$

Sub Problem 1a&

Max.

$Z = x_1 + x_2$

S/t

$2x_1 + 5x_2 \leq 16$

$6x_1 + 5x_2 \leq 30$

$x_2 \leq 1$

$x_1 \leq 4$

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Sub Problem 1b

Max

$$Z = x_1 + x_2$$

S/t

$$2x_1 + 5x_2 \leq 16$$
$$6x_1 + 5x_2 \leq 30$$
$$x_2 \leq 1$$
$$x_1 \geq 5$$

First we solve the sub problem 1a

Consider:

$$x_1 \leq 4$$
$$x_1 + S_2 = 4$$
$$\Rightarrow x_1 = 4 - S_2$$

Consider, x_1 - Eq from optimal table of the sub problem 1

$$x_1 + \frac{1}{6}x_4 - \frac{5}{6}S_1 = \frac{25}{6}$$

$$4 - S_2 + \frac{1}{6}x_4 - \frac{5}{6}S_1 = \frac{25}{6}$$

$$\Rightarrow S_2 - \frac{1}{6}x_4 + \frac{5}{6}S_1 = 4 - \frac{25}{6}$$

$$\Rightarrow S_2 - \frac{1}{6}x_4 + \frac{5}{6}S_1 = -\frac{1}{6}$$

Adding this to optimal table of the sub problem 1.

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B	x_1	x_2	x_3	x_4 ↓	S_1	S_2	Sol.
Z	0	0	0	1/6	1/6	0	31/6
x_2	0	1	0	0	1	0	1
x_1	1	0	0	1/6	-5/6	0	25/6
x_3	0	0	1	-1/3	-10/3	0	8/3
← S_2	0	0	0	(-1/6) P.E	5/6	1	-1/6
Ratio				|1/6 / -1/6|		
				= 1			

B	x_1	x_2	x_3	x_4	S_1	S_2	Sol.
Z	0	0	0	0	1	1	5
x_2	0	1	0	0	1	0	1
x_1	1	0	0	0	0	1	4
x_3	0	0	1	0	-5	-2	3
x_4	0	0	0	1	-5	-6	1

This is the optimal table of sub P.1a.

$Z = 5$; $x_1 = 4$, $x_2 = 1$

Now, we solve the sub problem 1b.

Consider:

$$x_1 \geq 5$$

$$-x_1 \geq -5$$

$$x_1' = 5 + S_3$$

Consider x_1 - Eq from optimal table of sub problem 1.

$$x_1 + \frac{1}{6}x_4 - \frac{5}{6}S_1 = \frac{25}{6}$$

$$5 + S_3 + \frac{1}{6}x_4 - \frac{5}{6}S_1 = \frac{25}{6}$$

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$$\Rightarrow S_3 + \frac{1}{6}x_4 - \frac{5}{6}S_1 = -\frac{5}{6}.$$

Adding this cell to the optimal table of sub problem 1.

B	x_1	x_2	x_3	x_4	S_1	S_3	Sol.
Z	0	0	0	1/6	1/6	0	31/6
x_2	0	1	0	0	1	0	1
x_1	1	0	0	1/6	-5/6	0	25/6
x_3	0	0	1	-1/3	-10/3	0	8/3
← S_3	0	0	0	1/6	(-5/6) P.E	1	-5/6
Ratio					$\left\lvert\frac{1/6}{-5/6}\right\rvert$ = 1/5		

B	x_1	x_2	x_3	x_4	S_1	S_3	Sol.
Z	0	0	0	1/5	0	1/5	5
x_2	0	1	0	1/5	0	6/5	0
x_1	1	0	0	0	0	-1	5
x_3	0	0	1	-1	0	-4	6
S_1	0	0	0	-1/5	1	-6/5	1

This is optimal table of sub problem 1b.

$Z = 5$, $x_1 = 5$; $x_2 = 0$.

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Now, we solve the sub problem 2.

Consider:

$$x_2 \geq 2$$

$$-x_2 \leq -2$$

$$-x_2 + S_4 = -2$$

$$\Rightarrow x_2 = 2 + S_4.$$

Consider:

x_2 = Eq. from optimal table of the given problem

$$x_2 + \frac{3}{10}x_3 - \frac{1}{10}x_4 = \frac{9}{5}$$

$$2 + S_4 + \frac{3}{10}x_3 - \frac{1}{10}x_4 = \frac{9}{5}$$

$$\Rightarrow S_4 + \frac{3}{10}x_3 - \frac{1}{10}x_4 = -\frac{1}{5}$$

Adding this cut to the optimal table of given problem.

B	x_1	x_2	x_3	x_4 ↓	S_4	Sol.
Z	0	0	1/20	3/20	0	53/10
x_2	0	1	3/10	-1/10	0	9/5
x_1	1	0	-1/4	1/4	0	7/2
← S_4	0	0	3/10	(-1/10) PE	1	-1/5
Ratio.				3/20 / -1/10 = 3/2		

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B	x_1	x_2	x_3	x_4	S_4	Sol.
Z	0	0	1/2	0	3/2	5
x_2	0	1	0	0	-1	2
x_1	1	0	1/2	0	5/2	3
x_4	0	0	-3	1	-10	2.

This is the optimal table of the sub problem 2.

$Z = 5$; $x_1 = 3$; $x_2 = 2$.

Sol:

The problem has alternative sol. as.

(4, 1) , (5, 0) , (3, 2) and $Z = 5$.

Ans

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Section - II

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